CT Convolution (1A)

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8/18/14

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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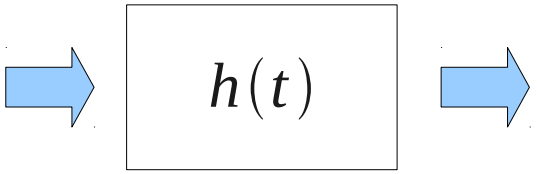
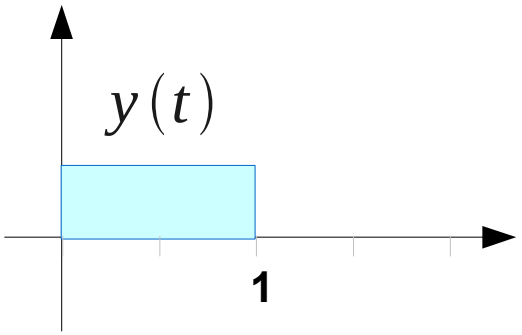
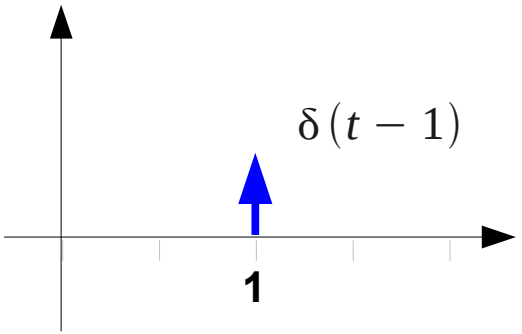
Convolution: delayed response of h(t) (1)

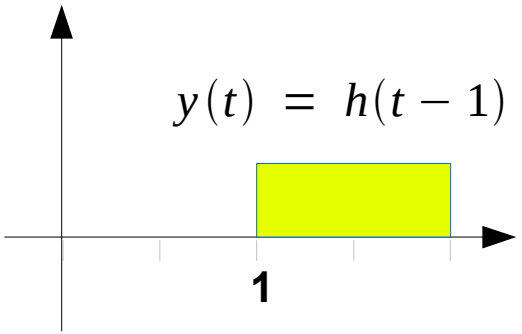
Impulse response

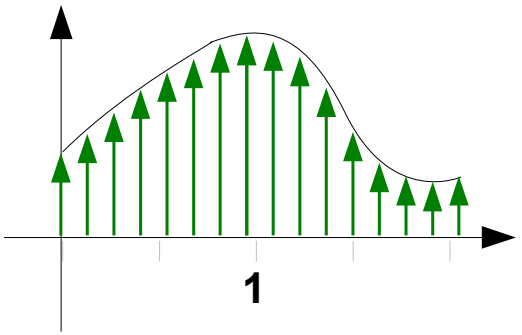


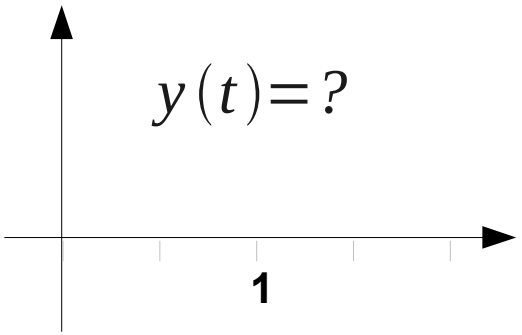




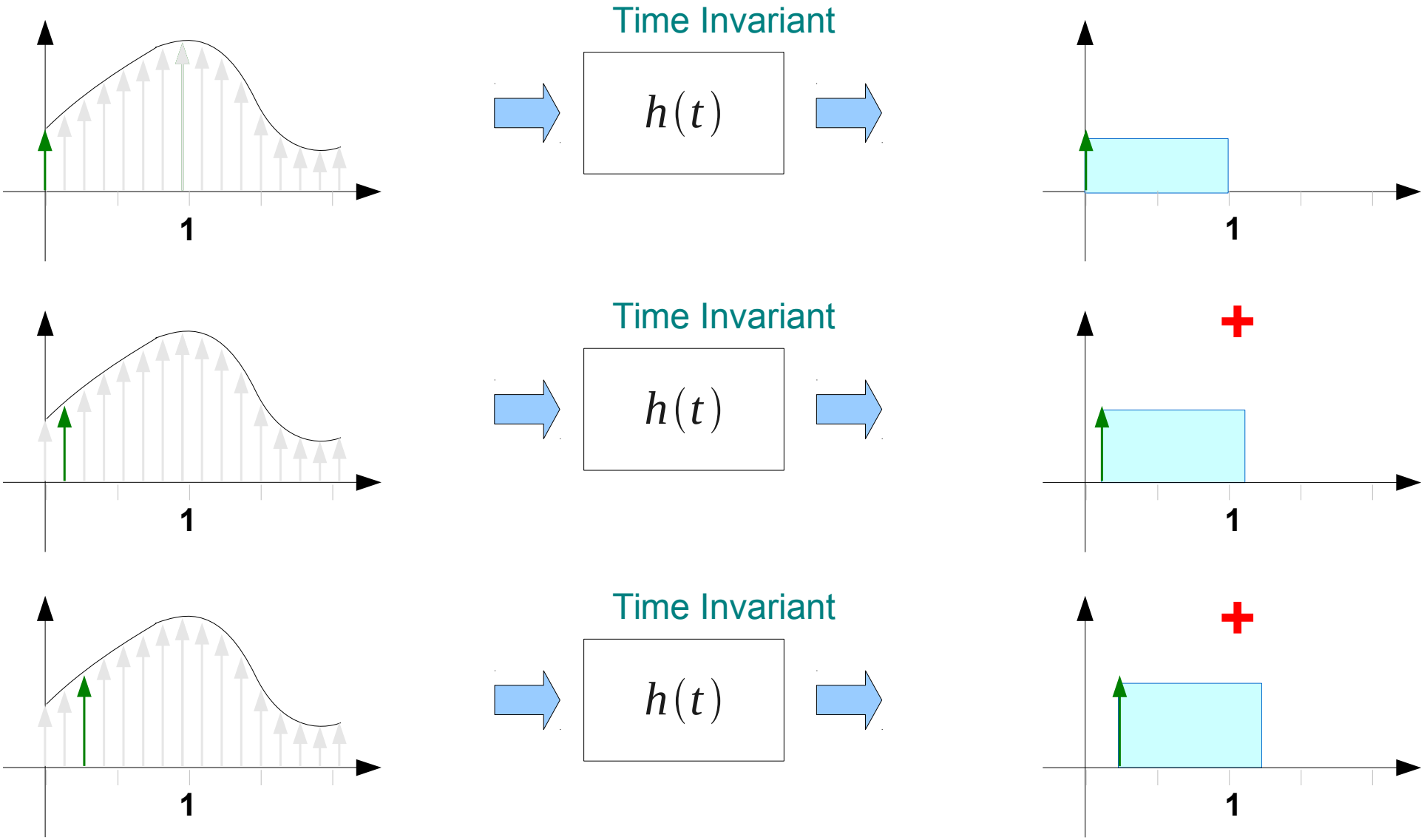
delayed response
by 1



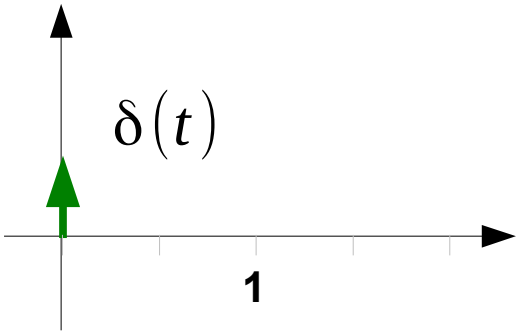




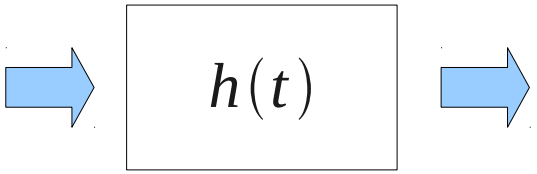
Convolution: delayed response of h(t) (2)

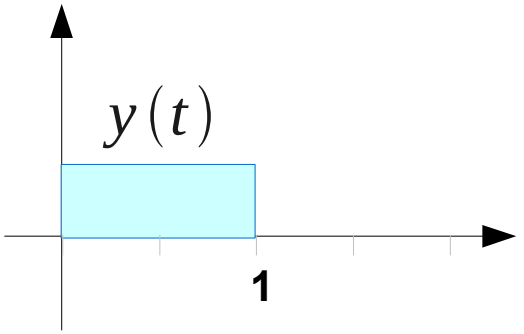


Convolution: delayed response of h(t) (3)



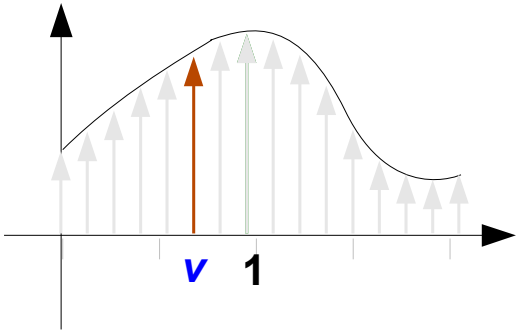
Linear System



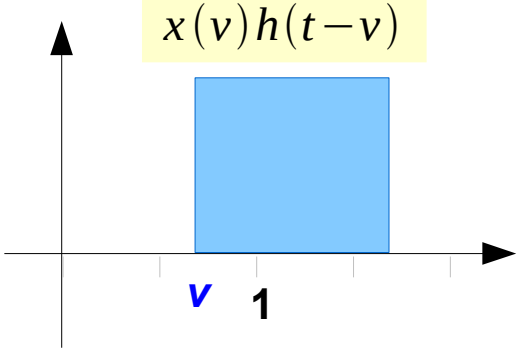


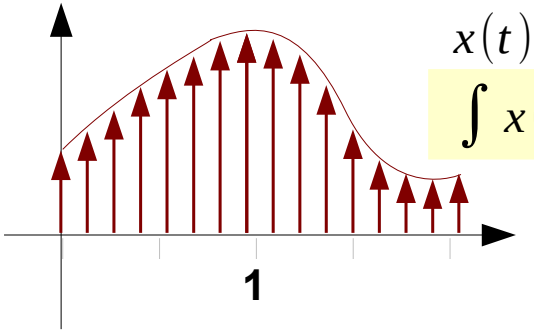
$$\delta(t-v) \Rightarrow h(t-v)$$

$$x(v)\,\delta(t-v) \Rightarrow x(v)\,h(t-v)$$



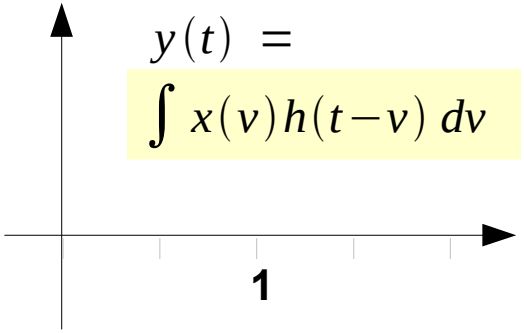
input value at time v
→ x(v)
delayed impulse response
→ x(v) h(t - v)



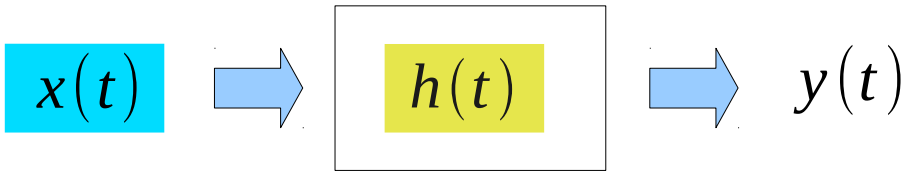


$$x(t) = \int x(v)\,\delta(t-v)\,dv$$

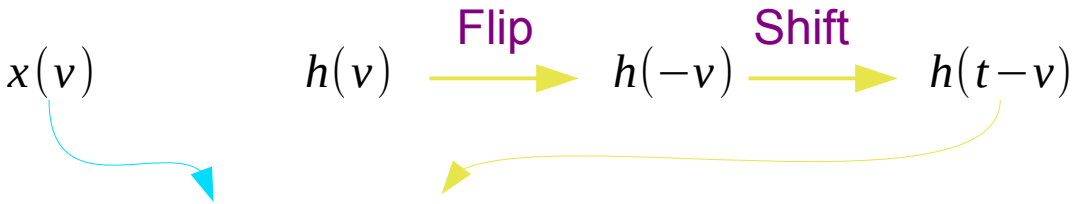
$$y(t) = \int x(v)\,h(t-v)\,dv$$



Convolution: Commutative Law

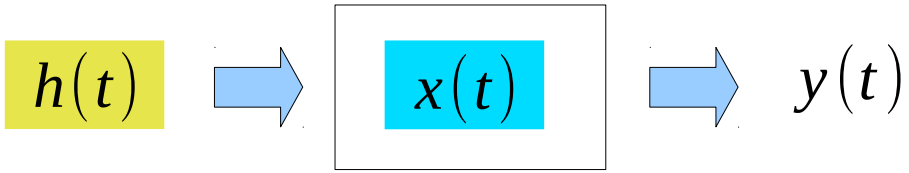


LTI



$$\int x(v)\,h(t-v)\,dv = y(t)$$

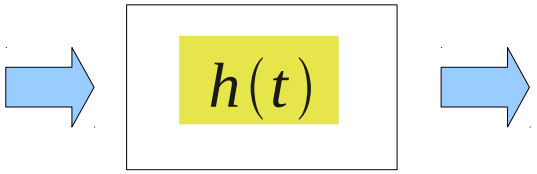
$$\int h(v)\,x(t-v)\,dv = y(t)$$

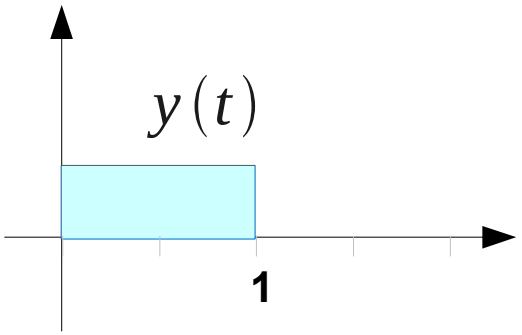


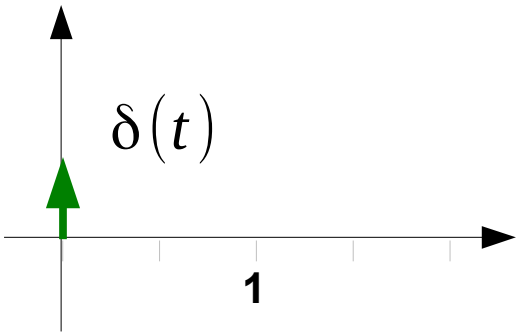
LTI

Convolution: delayed response of x(t) (1)

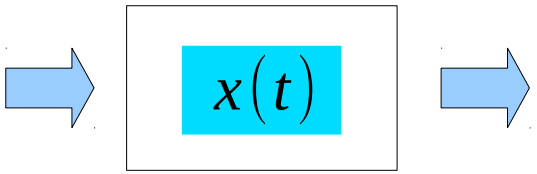
Impulse response ***h(t)***

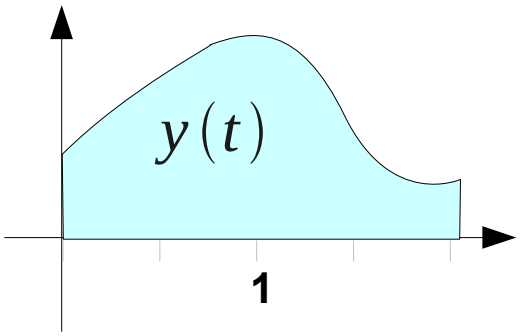


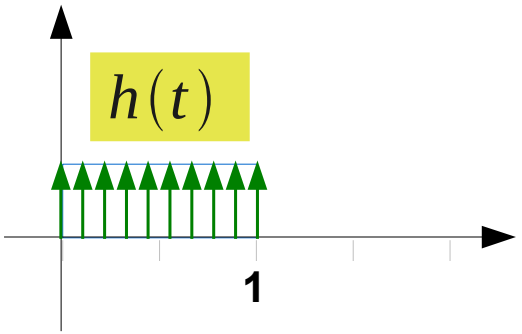




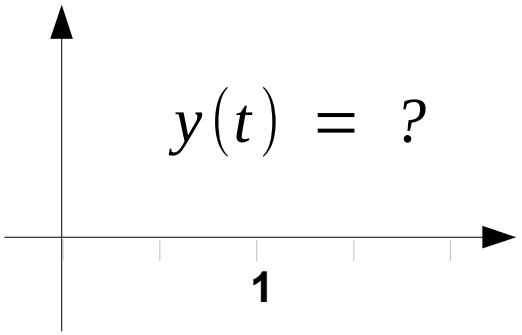
Impulse response ***x(t)***



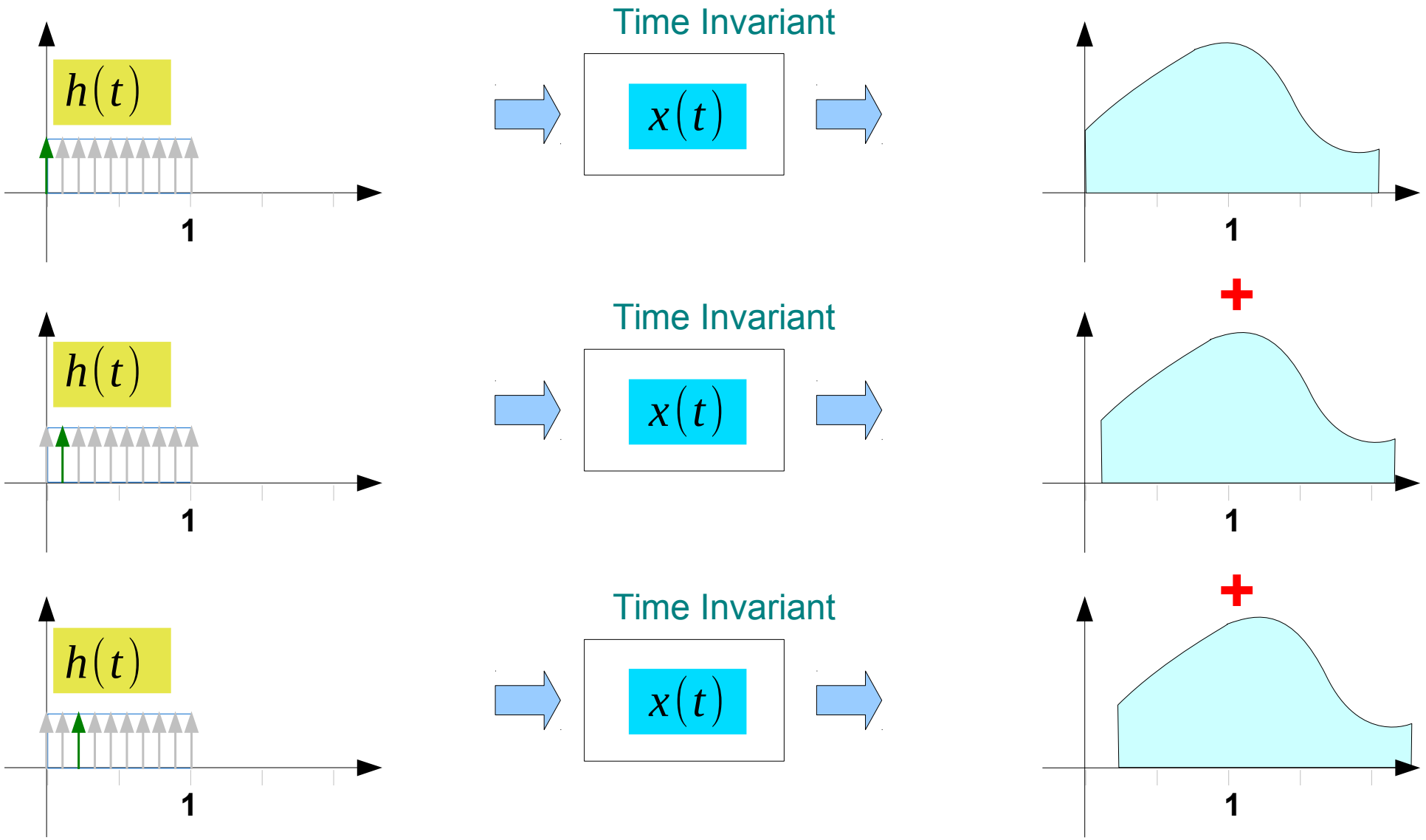




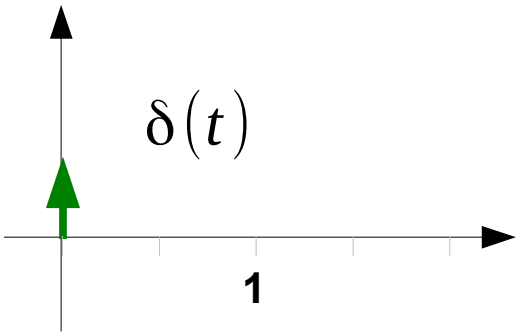
delayed response
by 1



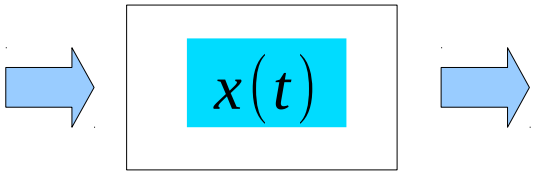
Convolution: delayed response of x(t) (2)

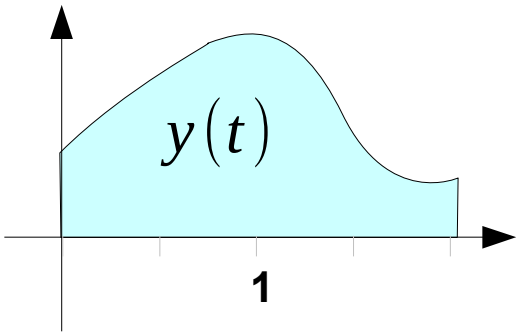


Convolution: delayed response of x(t) (3)



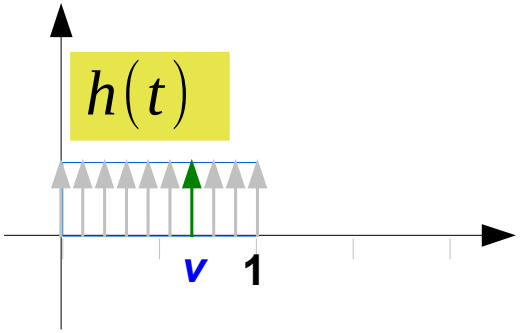
Linear System



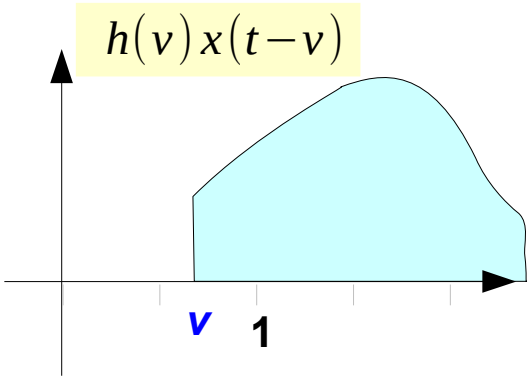


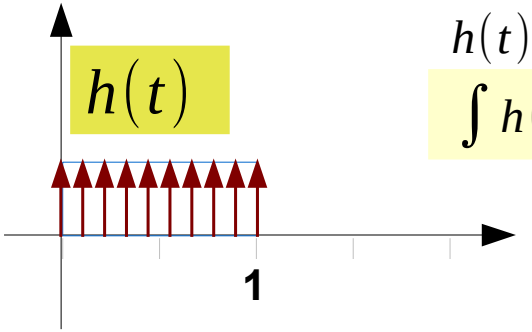
$\delta(t-v) \Rightarrow x(t-v)$

$h(v)\,\delta(t-v) \Rightarrow h(v)\,x(t-v)$



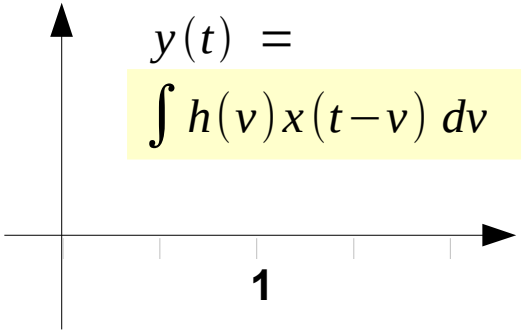
input value at time v
→ h(v)
delayed impulse response
→ h(v) x(t - v)



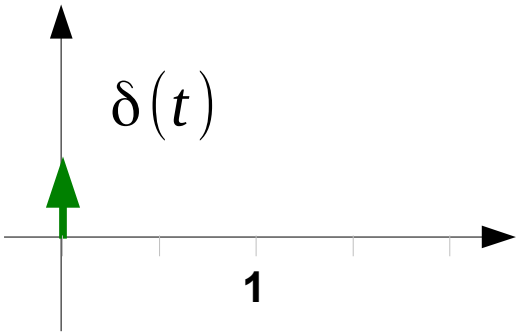


$$h(t) = \int h(v)\,\delta(t-v)\,dv$$

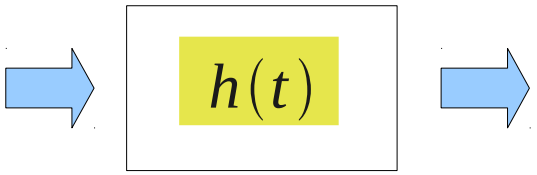
$$y(t) = \int h(v)\,x(t-v)\,dv$$

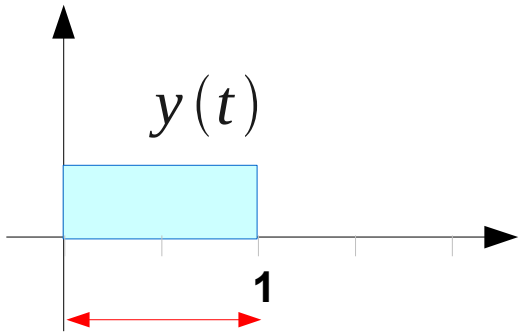


FIR and IIR



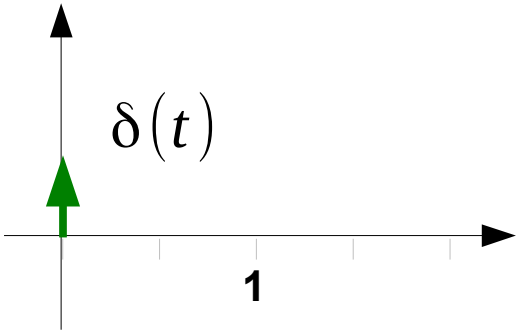
Impulse response ***h(t)***



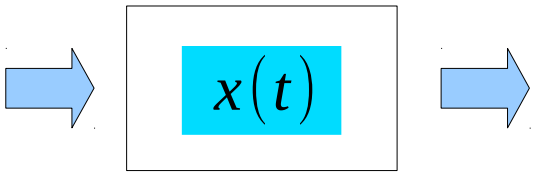


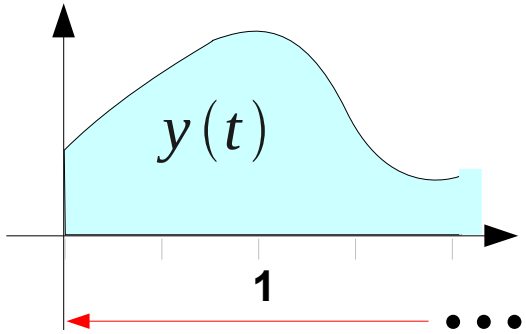
Finite Impulse Response

Finite Duration



Impulse response ***x(t)***

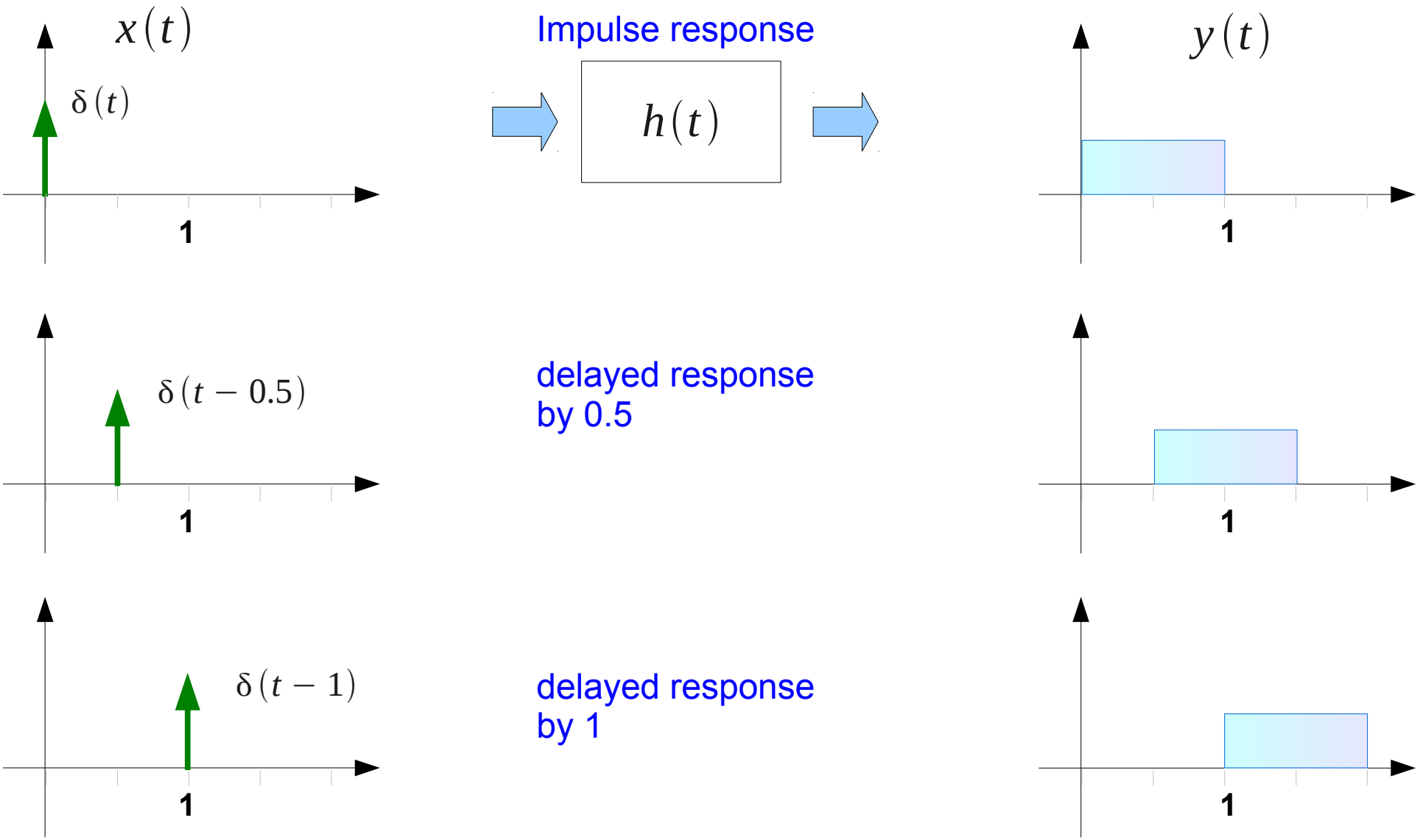




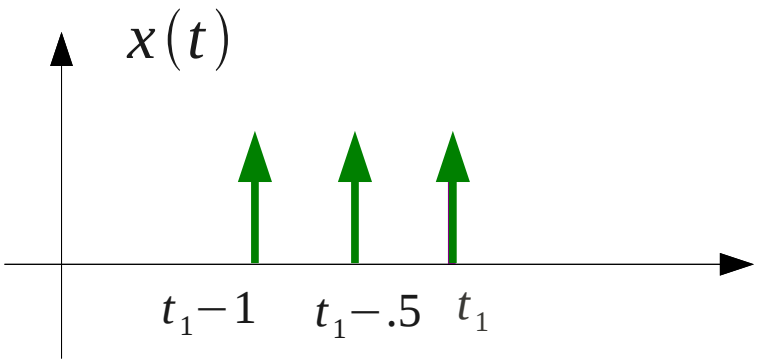
Infinite Impulse Response

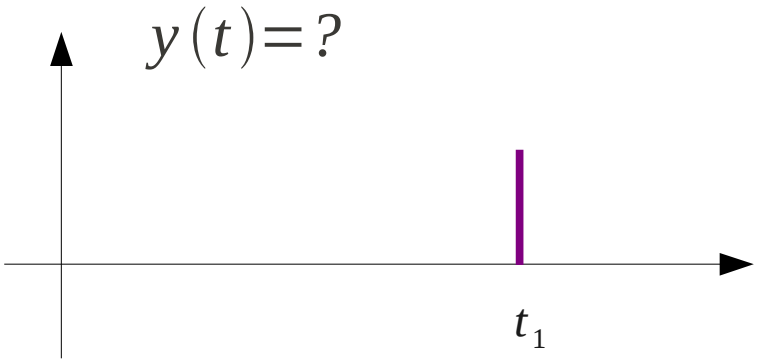
Infinite Duration

Impulse Response



LTI System





$\delta(t-t_1+1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+1)$

$\delta(t-t_1+.5) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+.5)$

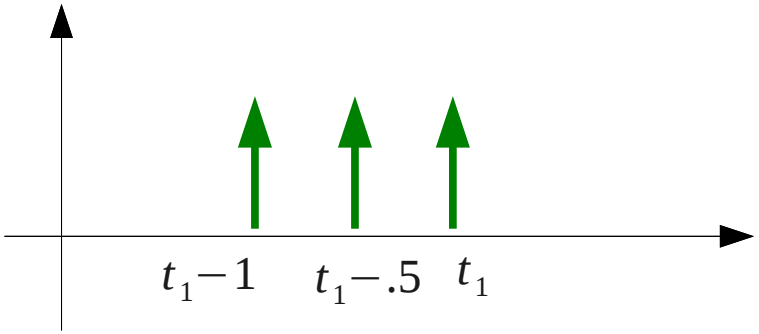
$\delta(t-t_1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1)$

$\rightarrow \boxed{h(t)} \rightarrow$

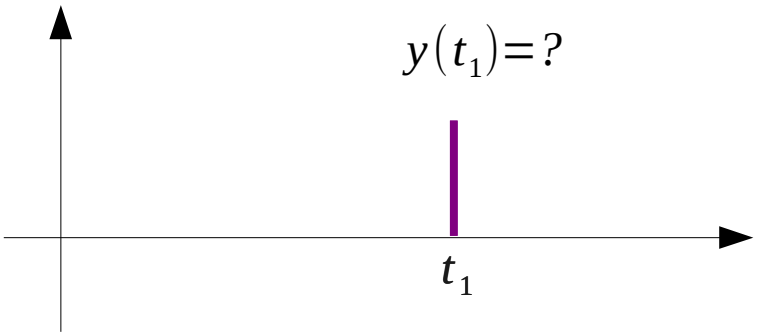
$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$

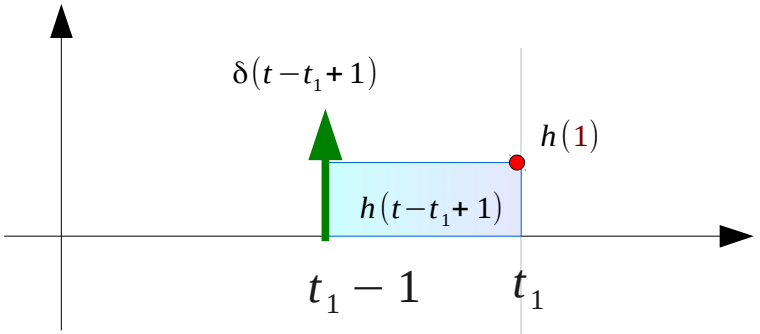
$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$

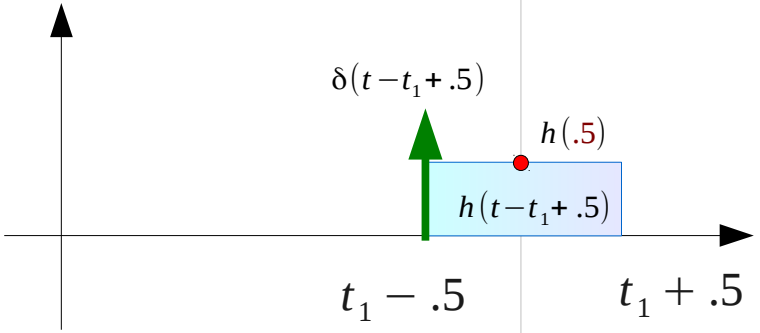
Computing $y(t_1)$: Output at $t = t_1$

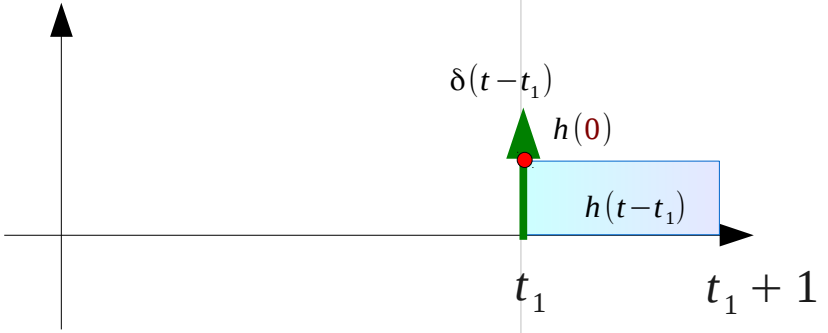


$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$



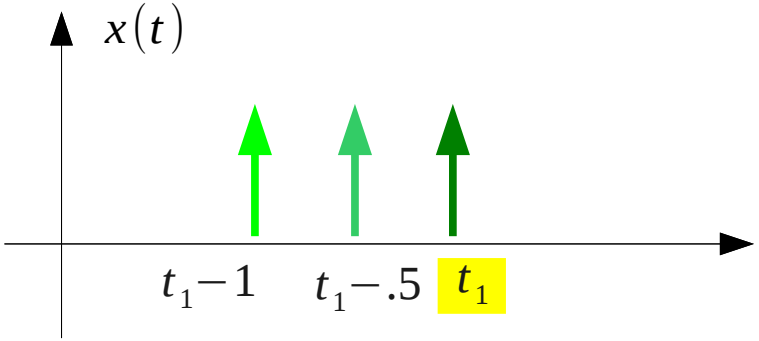


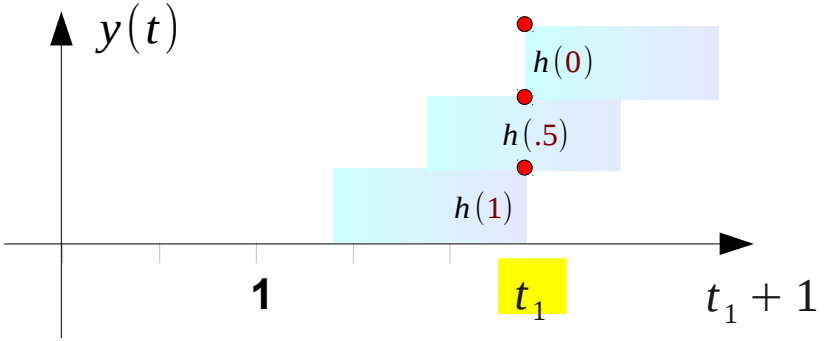




Computing $y(t_1)$: delayed impulse response h(t)

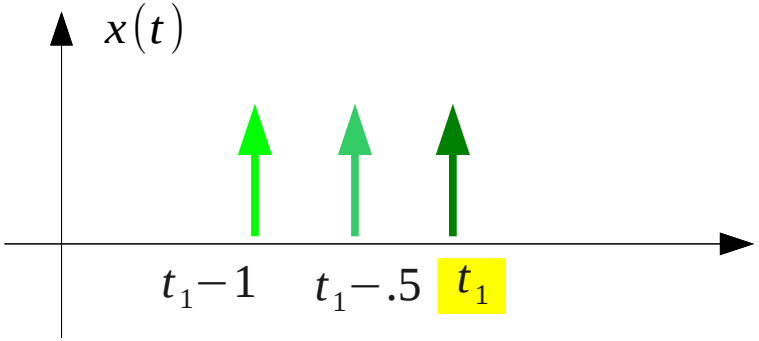
delayed impulse response – h(t)

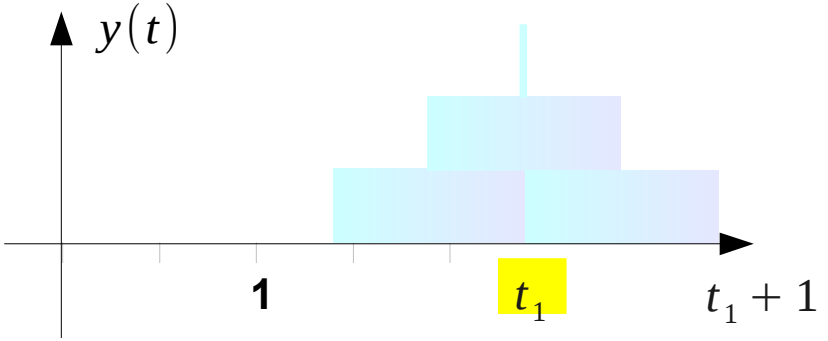




$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$





$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Computing $y(t_1)$: flip and shift $x(t)$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $\quad t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and then shift to the right by t $\quad v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v)\, h(v)\, dv$$

$$= \int \delta(t-v-t_1+1)\, h(v)\, dv \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(t-v-t_1+.5)\, h(v)\, dv \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(t-v-t_1)\, h(v)\, dv \rightarrow h(t-t_1)$$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: flip and shift $h(t)$

$h(t)$

Change of variables $t \rightarrow v$

$h(v)$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$h(t-v)$

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(v-t_1)h(t-v)\,dv \rightarrow h(t-t_1)$$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: commutativity (1)

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1+.5)$$

$$+ \int \delta(v-t_1)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1)$$

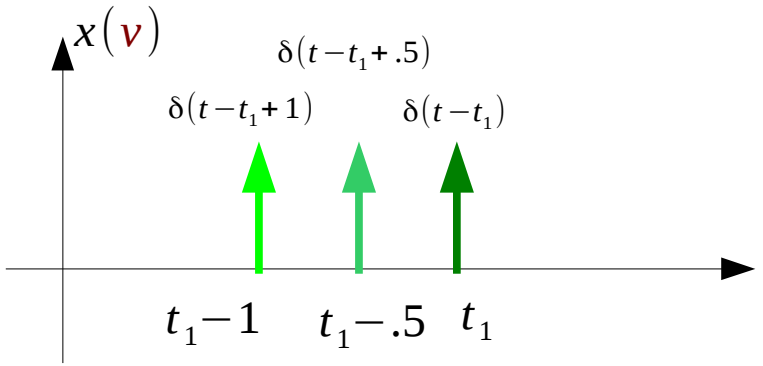
$$y(t) = \int x(t-v)h(v)\,dv$$

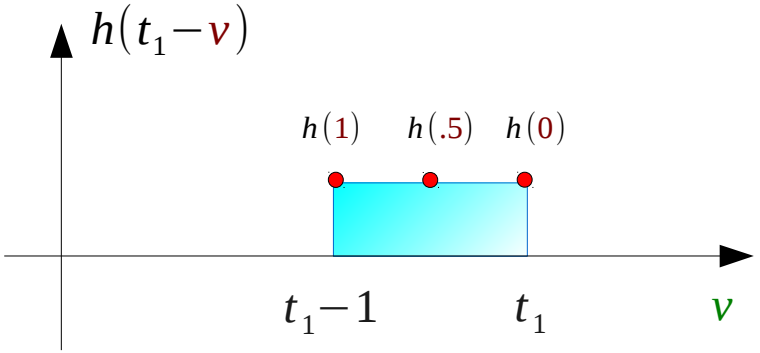
$$= \int \delta(t-v-t_1+1)h(v)\,dv \quad \Leftarrow \quad h(t-t_1+1)$$

$$+ \int \delta(t-v-t_1+.5)h(v)\,dv \quad \Leftarrow \quad h(t-t_1+.5)$$

$$+ \int \delta(t-v-t_1)h(v)\,dv \quad \Leftarrow \quad h(t-t_1)$$

Flip and Shift h(t)





Computing $y(t_1)$: commutativity (2)

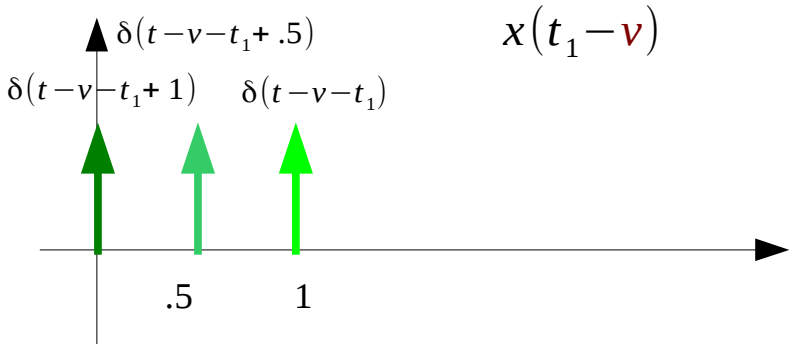
$$y(t) = \int x(v)h(t-v)\,dv$$

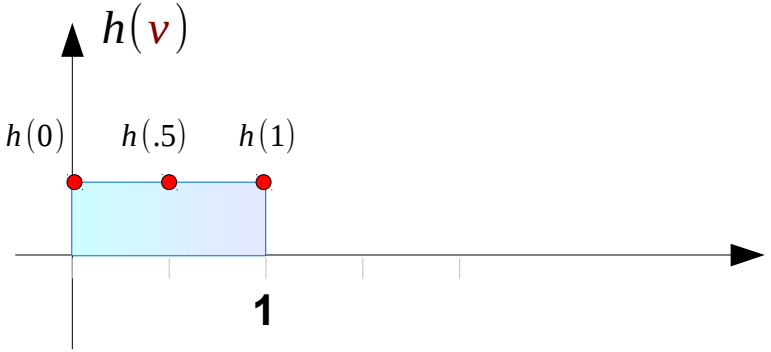
$$= \int \delta(v-t_1+1)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1+1) \quad \Leftarrow \quad = \int \delta(t-v-t_1+1)h(v)\,dv$$

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1+.5) \quad \Leftarrow \quad + \int \delta(t-v-t_1+.5)h(v)\,dv$$

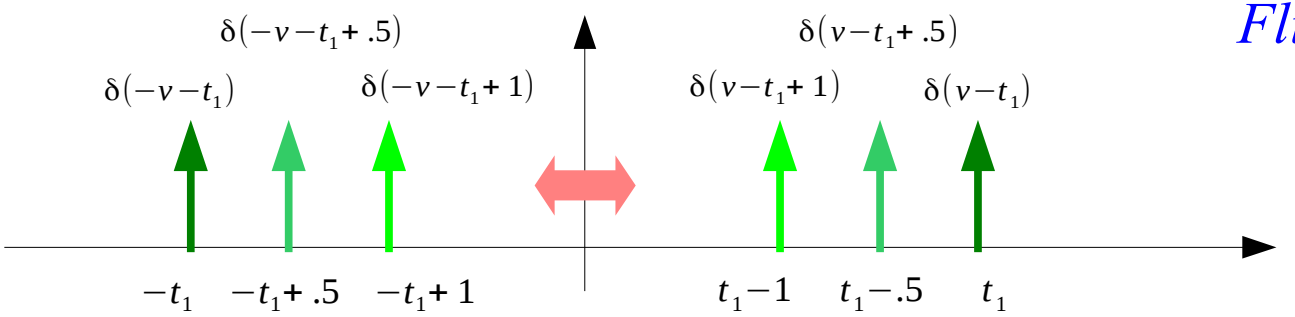
$$+ \int \delta(v-t_1)h(t-v)\,dv \quad \Rightarrow \quad h(t-t_1) \quad \Leftarrow \quad + \int \delta(t-v-t_1)h(v)\,dv$$

Flip and shift input x(t)



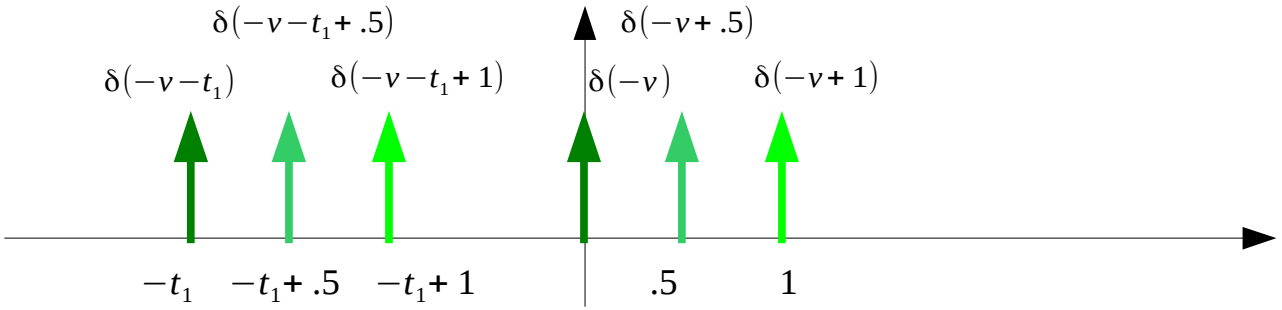


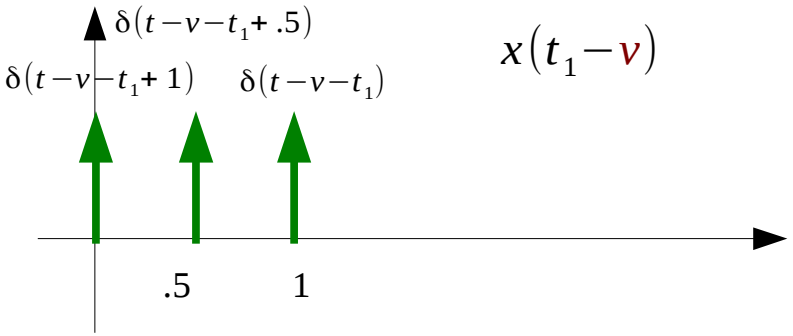
Computing $y(t_1)$: commutativity (3)



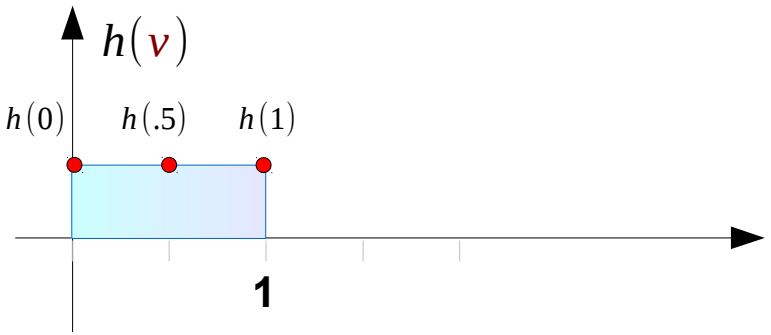
Flip around y-axis

Shift to the right by t_1

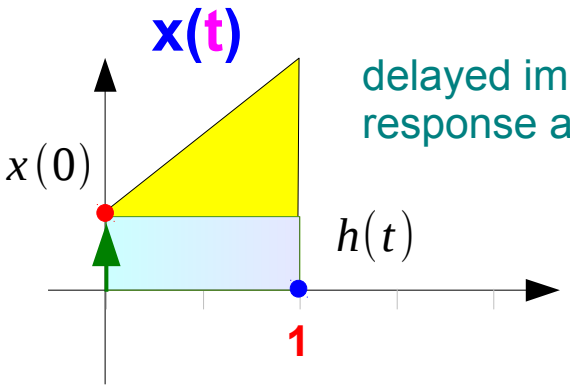




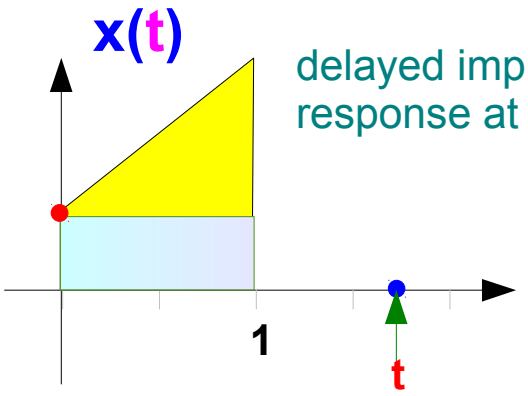
$x(t_1 - v)$



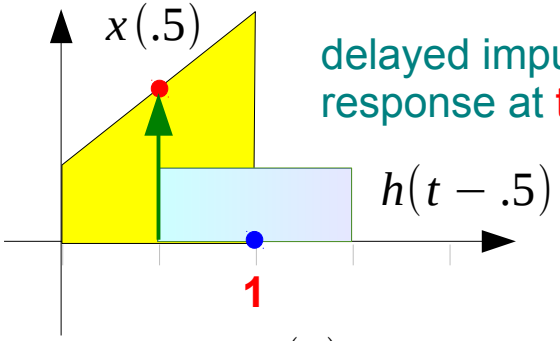
Computing y(1), y(t)



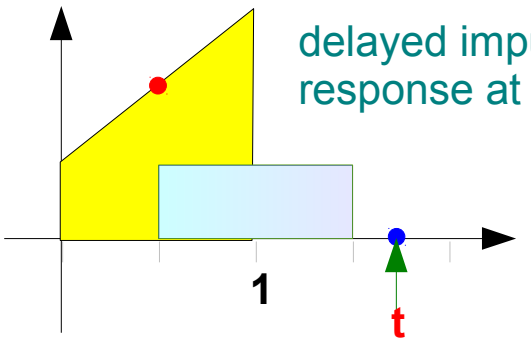
delayed impulse response at t=1 $x(0)h(1)$



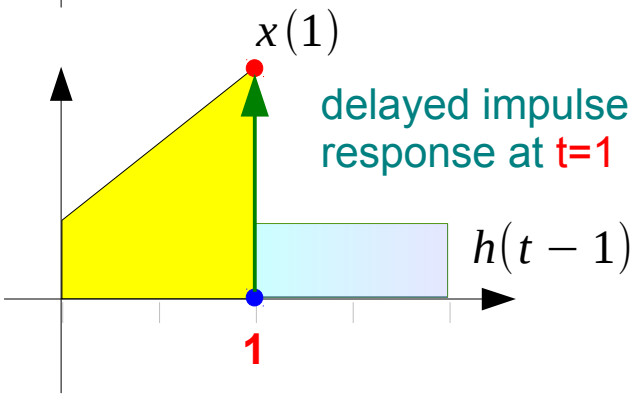
delayed impulse response at t $x(0)h(t)$



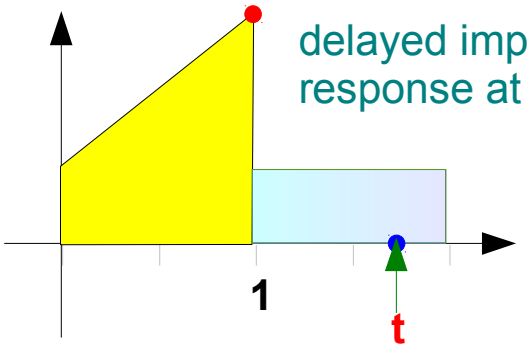
delayed impulse response at t=1 $x(.5)h(.5)$



delayed impulse response at t $x(.5)h(t-.5)$

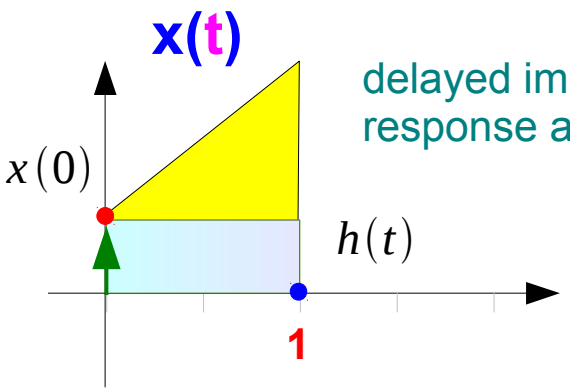


delayed impulse response at t=1 $x(1)h(0)$



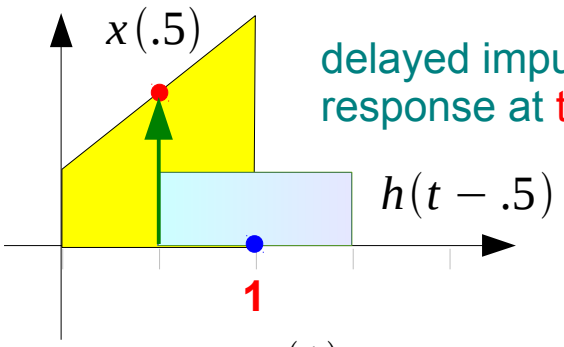
delayed impulse response at t=1 $x(1)h(t-1)$

Computing y(1) : shift & flip x(t)



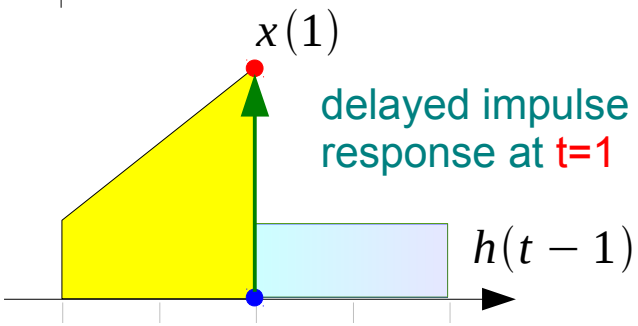
delayed impulse response at t=1

$x(0)h(1)$



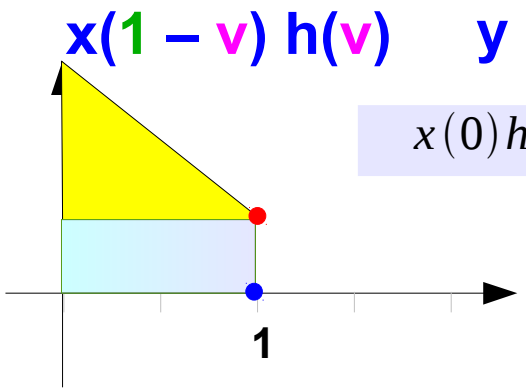
delayed impulse response at t=1

$x(.5)h(.5)$

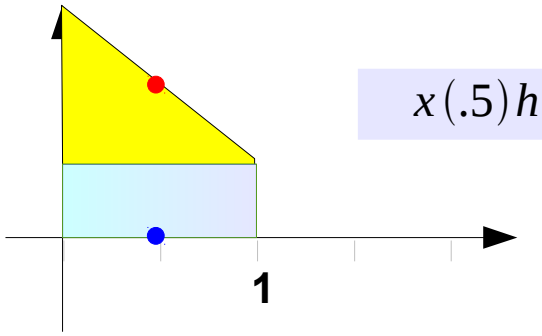


delayed impulse response at t=1

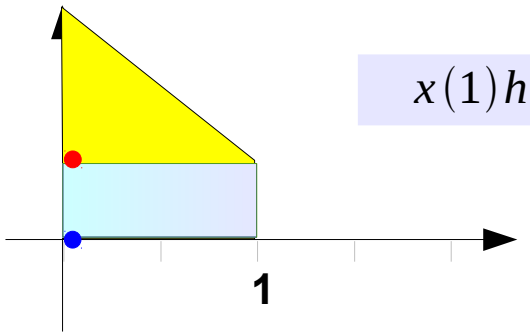
$x(1)h(0)$



$x(0)h(1)$



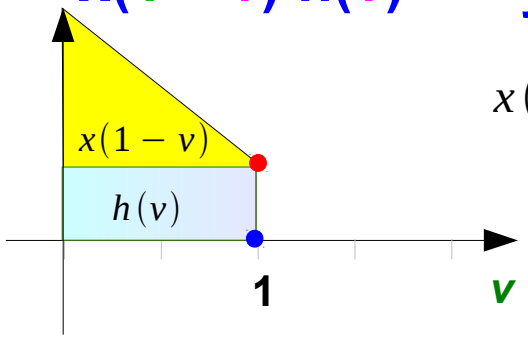
$x(.5)h(.5)$



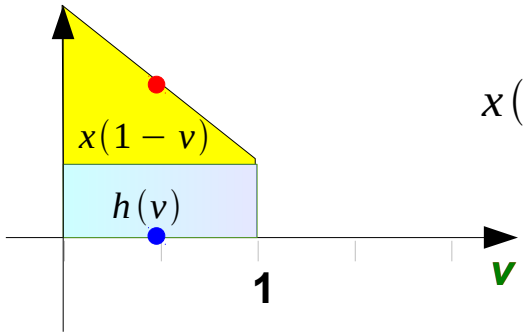
$x(1)h(0)$

Computing y(t) : shift & flip x(t)

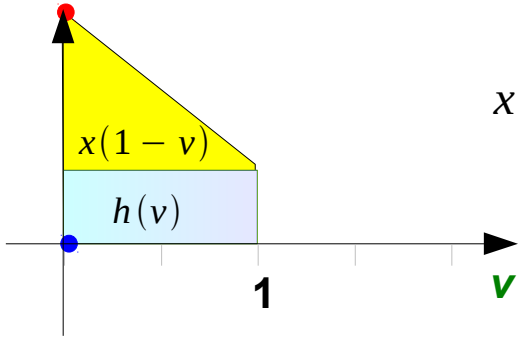
x(1 – v) h(v) **y (1)**



$$x(0)h(1)$$

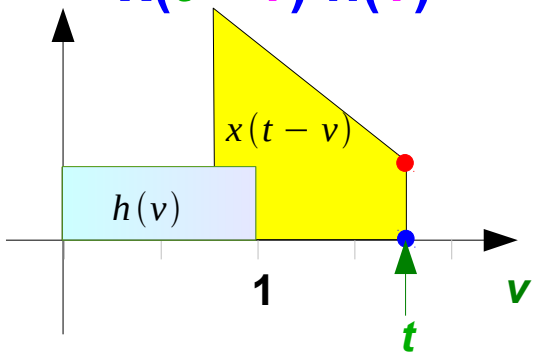


$$x(.5)h(.5)$$

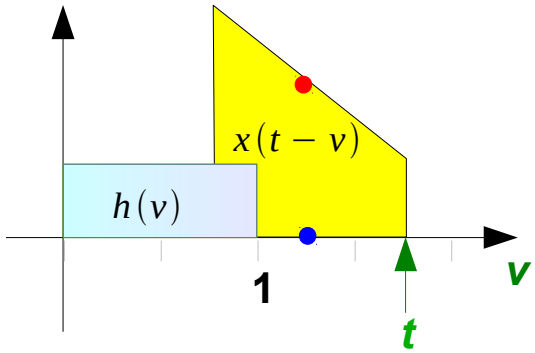


$$x(1)h(0)$$

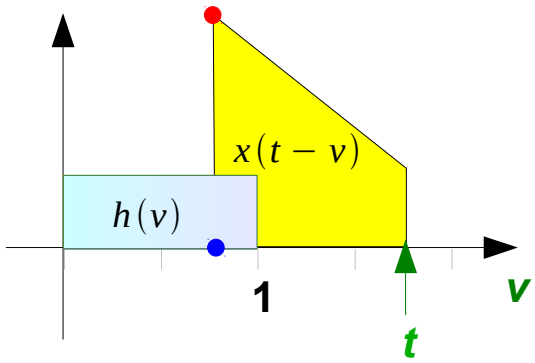
x(t – v) h(v) **y (t)**



$$x(t-t)h(t)$$
$$= x(0)h(t)$$

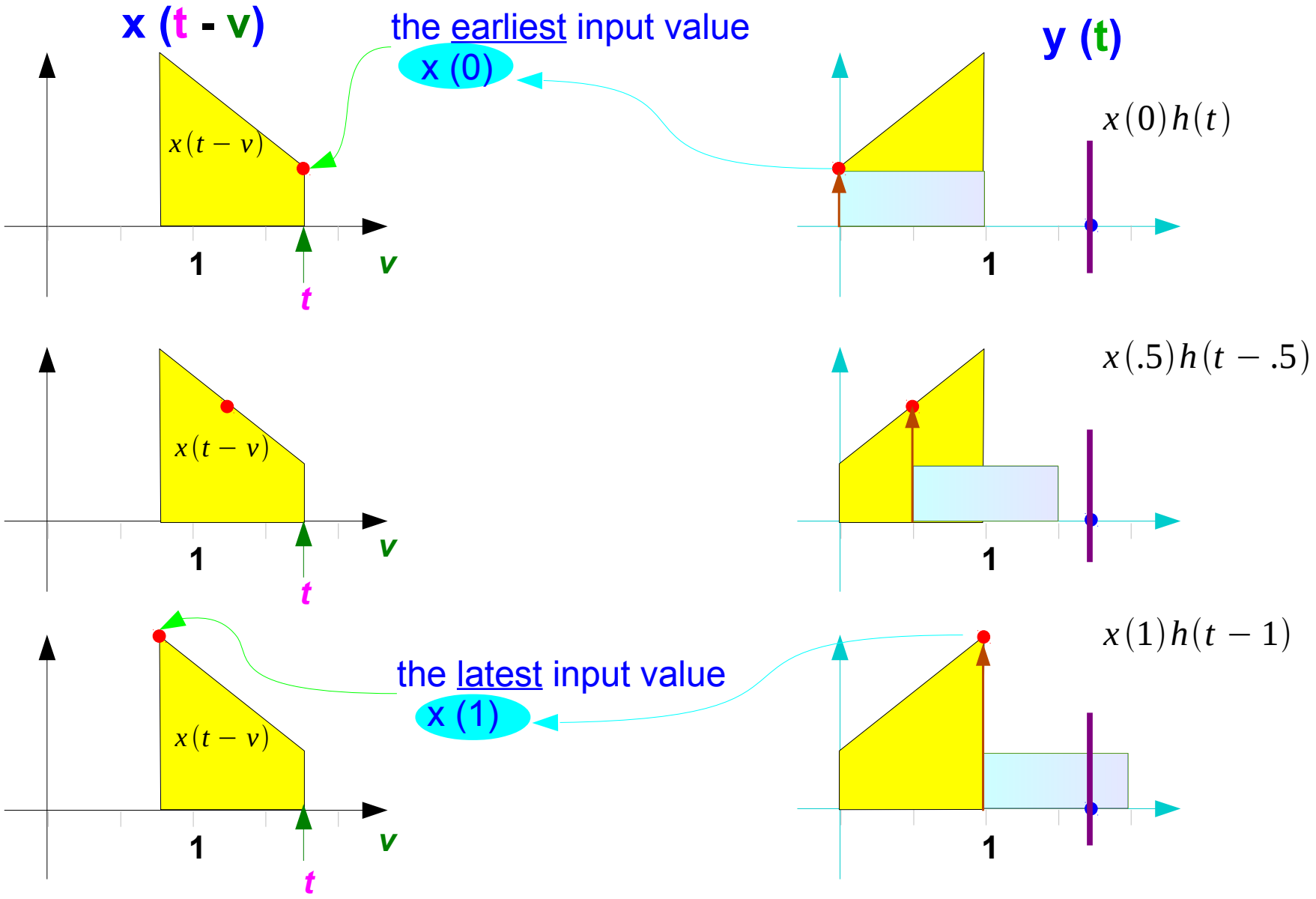


$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$

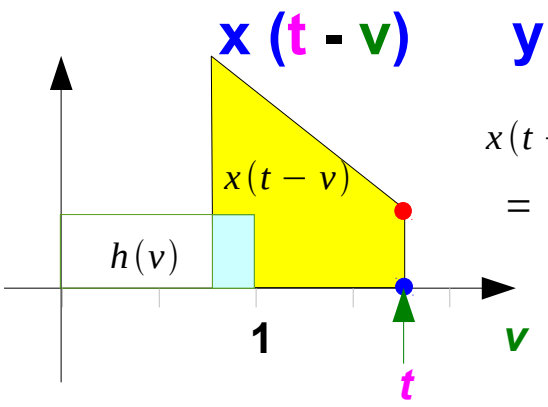


$$x(t-t+1)h(t-1)$$
$$= x(1)h(t-1)$$

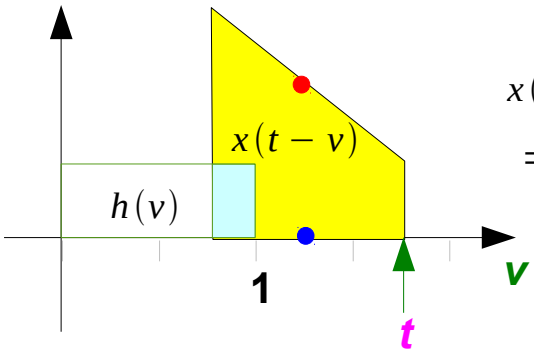
Computing y(t) : the earliest and latest inputs



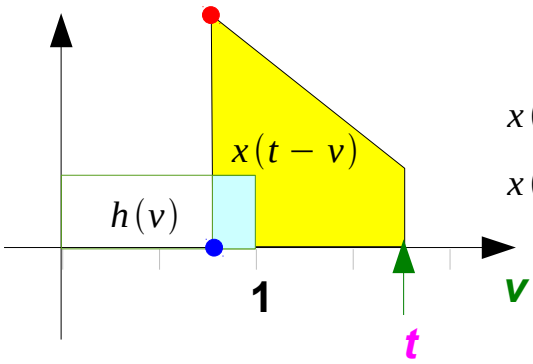
Computing y(t), y(s)



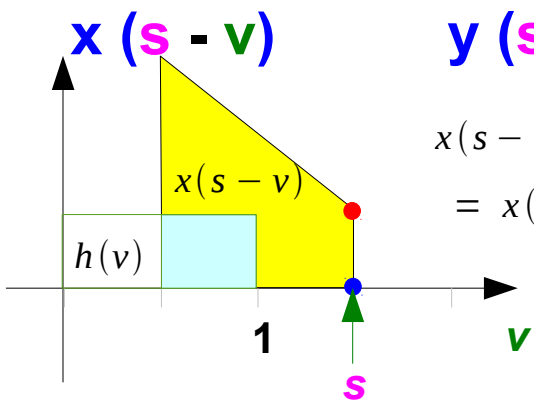
$$x(t-t)h(t)$$
$$= x(0)h(t)$$



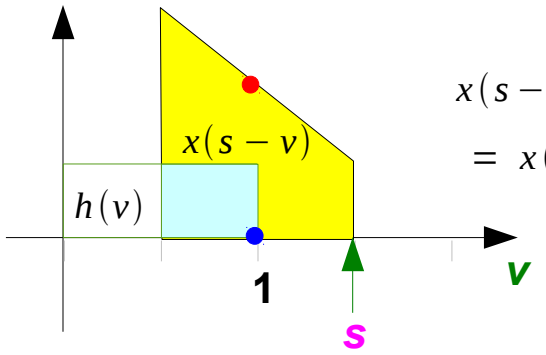
$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$



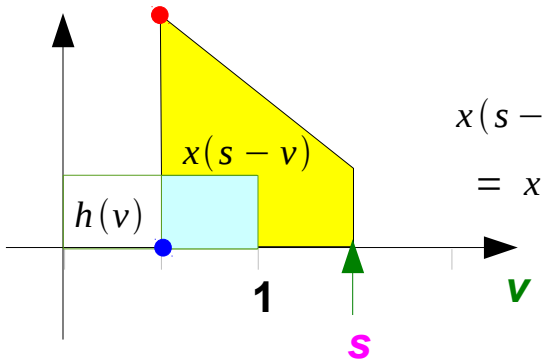
$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$



$$x(s-s)h(s)$$
$$= x(0)h(s)$$



$$x(s-s+.5)h(s-.5)$$
$$= x(.5)h(s-.5)$$

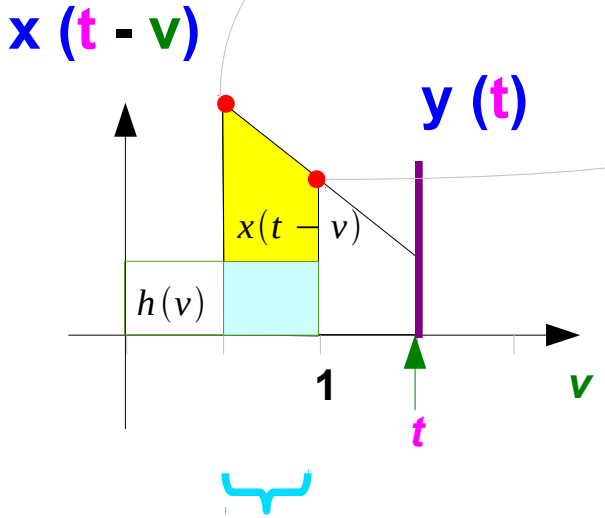


$$x(s-s+1)h(s-1)$$
$$= x(1)h(s-1)$$

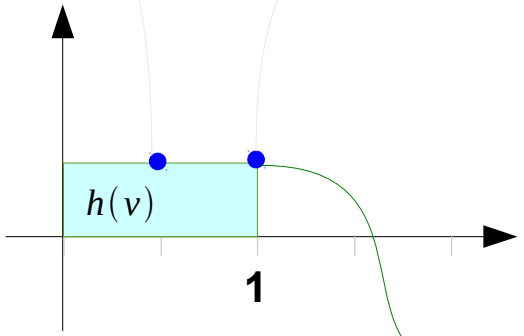
Computing y(t) : earliest and latest inputs

the latest input value
that affects the output value y(t)

the earliest input value
that affects the output value y(t)

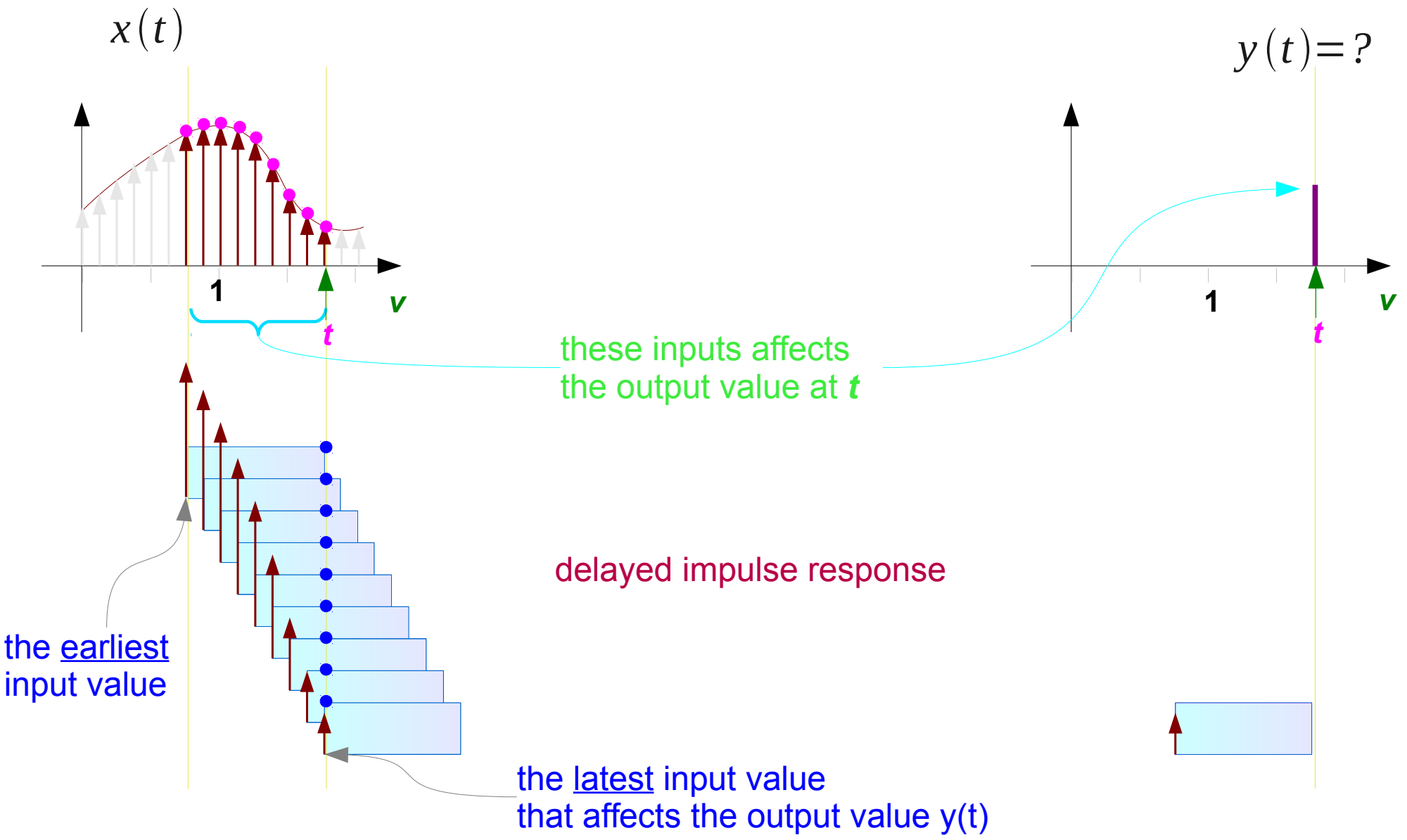


$$y(t) = \int x(t-v)h(v)\,dv$$

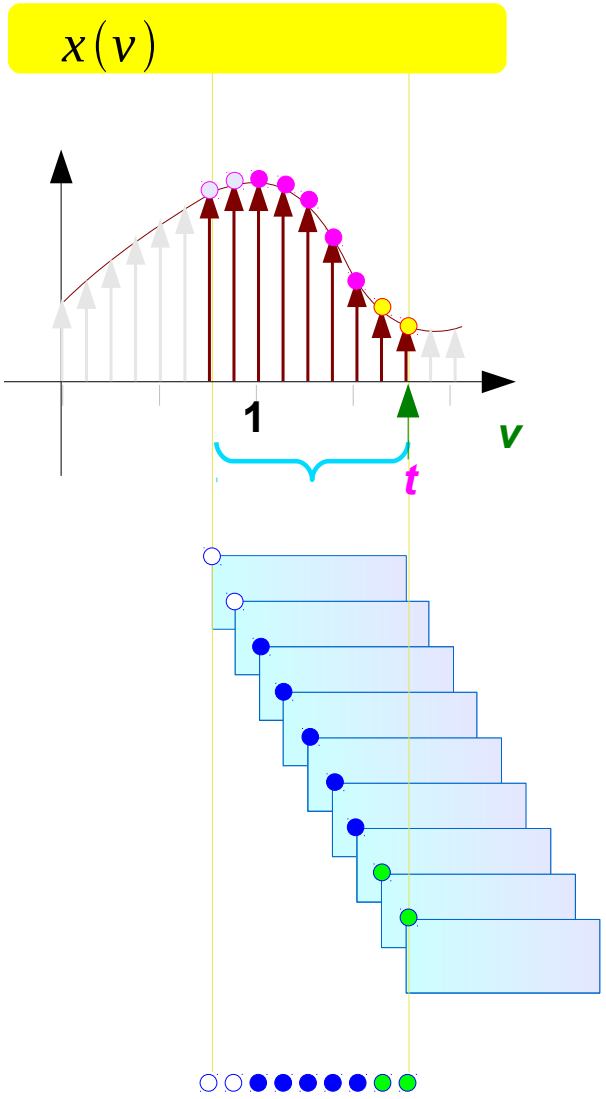


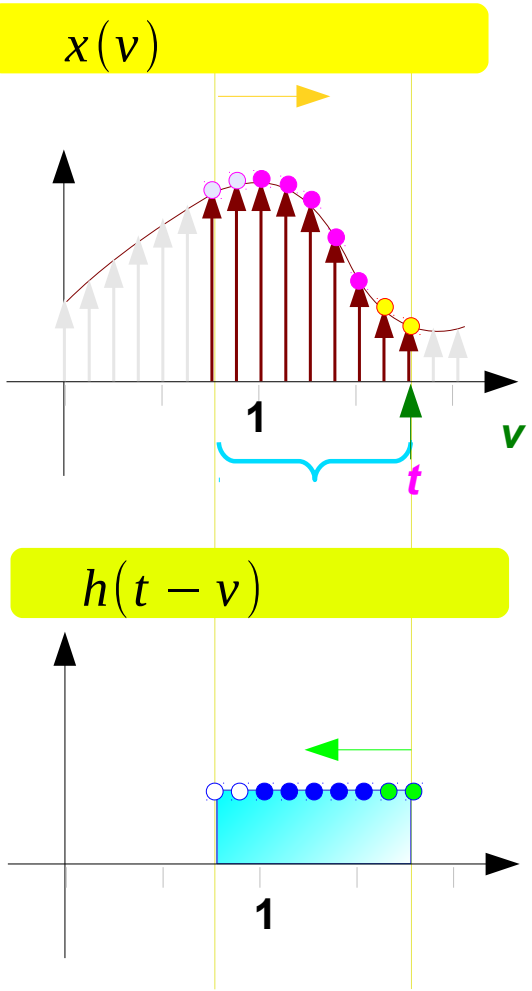
this value of h(t)
is used first

Computing y(t) : delayed impulse response

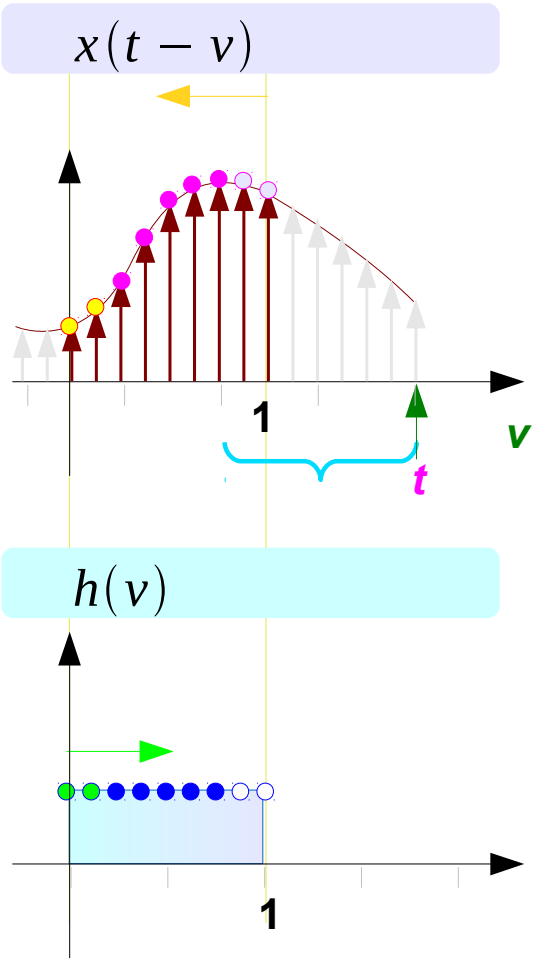


Computing y(t) : multiplication sequence





$$\int x(v)h(t-v)\,dv$$



$$\int x(t-v)h(v)\,dv$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003