# Background – Functions (1C)

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This document was produced by using LibreOffice.

# Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

# First-Class Functions

**first-class functions**

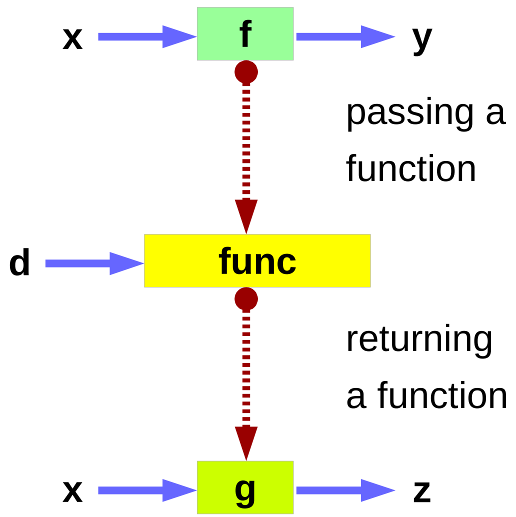
**functions** are treated as **first-class citizens**

the function names do not have any special status
they are treated like ordinary variables with a function type.

the language supports

- **passing** functions **as arguments** to other functions,
- **returning** functions **as the values** from other functions,
- **assigning** functions to variables
- **storing** functions in data structures.

- supporting **anonymous functions** (function literals) as well




https://en.wikipedia.org/wiki/First-class_function

# Higher-Order and First order Functions

**first-class functions** are a necessity
in the **functional programming style**
where **higher**-**order functions** are widely used

A **higher-order function** is a function
that takes other functions as arguments
or returns a function as result.

A **first-order function** is a function
that does not takes other functions as arguments
nor returns a function as result.

https://en.wikipedia.org/wiki/First-class_function

# Higher-Order Function Example

A simple example of a **higher-order function**

the **map** function,

which takes a function and a list,

as its arguments,

returns the list formed

by applying the function

to each member of the list.

For a language to support **map**, (higher-ordered function)

it must support passing a function as an argument.

```
map (+3) [1, 2, 3]
[4, 5, 6]

(+3) :: a -> a
A function argument
```

https://en.wikipedia.org/wiki/First-class_function

# Functionals in mathematics

a **higher-order function**
(**functional**, **functional form** or **functor**)
is a function that does at least one of the following:

- takes one or more functions as arguments
  (i.e. procedural parameters),
- returns a function as its result.

All other functions are **first-order functions**.

https://en.wikipedia.org/wiki/Functor

# Functional Examples

In mathematics **higher-order functions** are also termed **operators** or **functionals**.

The **differential operator** in calculus is a common example, since it **maps** a function to its derivative, also a function.

$$(D^2-2D+1)f(x)$$

$$f''(x)-2f'(x)+f(x)$$

https://en.wikipedia.org/wiki/Functor

# Functors in mathematics

**Higher-order functions** should not be confused
with other uses of the word "**functor**" in mathematics

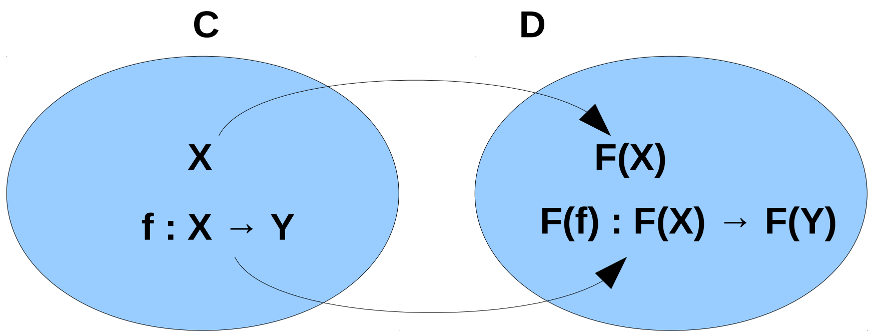
**a functor is a map between categories**

Let **C** and **D** be **categories**.

A **functor F** from **C** to **D** is a **mapping** that

associates to each **object X** in **C**
an object **F(X)** in **D**,

associates to each **morphism f : X → Y** in **C**
a morphism **F(f) : F(X) → F(Y)** in **D**




https://en.wikipedia.org/wiki/Functor

# Functors and morphism

Let **C** and **D** be **categories**.

A **functor F** from **C** to **D** is a **mapping** that
associates to each **object X** in **C**
an object **F(X)** in **D**,
associates to each **morphism f : X → Y** in **C**
a morphism **F(f) : F(X) → F(Y)** in **D**

such that the following two conditions hold:

$\mathbf{F(id_X) = id_{F(X)}}$ for every object **X** in **C**, — preserve **identity morphisms**

**F(g ∘ f) = F(g) ∘ F(f)** for all morphisms **f : X → Y** and **g : Y → Z** in **C**. — preseve **composition morphisms**

functors must preserve
**identity morphisms** and
**composition** of **morphisms**.

https://en.wikipedia.org/wiki/Functor

# Function Definition

**Function Definition I.**

```
square x = x * x
```

- **function type** is inferred → not efficient **Type Inference**

**Function Definition II.**

```
square :: Double -> Double
square x = x * x
```

– **function type declaration**

– **function definition**

- **function type declaration**
- **function definition**

http://www.toves.org/books/hsfun/

# Type Declaration

### Type Declaration

the declaration of an identifier's type

**identifier name :: type name** ...

identifier names (including function identifiers) must always begin with a lower-case letter

type names in Haskell always begin with a capital letter

http://www.toves.org/books/hsfun/

# Function Types and Type Classes

**Function Definition I.**

square x = x * x

**function definition**

=

**Function Definition II.**

square :: Double -> Double

square x = x * x

**function definition**

- function **type declaration**

=

**type class –** a set of types

- function **type** 1
- function **type** 2
- 
- function **type** n

Requirements

Subclasses

http://www.toves.org/books/hsfun/

# Curry & Uncurry

**f :: a -> b -> c** the curried form of **g :: (a, b) -> c**

**f = curry g**
**g = uncurry f**

**f x y = g (x,y)**

the curried form is usually more convenient because it allows partial application.

all functions are considered curried

all functions take just one argument

*the curried form*

**f :: a -> b -> c** ←currying— / —uncurrying→ **g :: (a, b) -> c**

**f x y**

**g (x, y)**

https://wiki.haskell.org/Currying

# Functions : First-class Data Types

functions are **first-class data types**

Haskell treats functions as regular data,

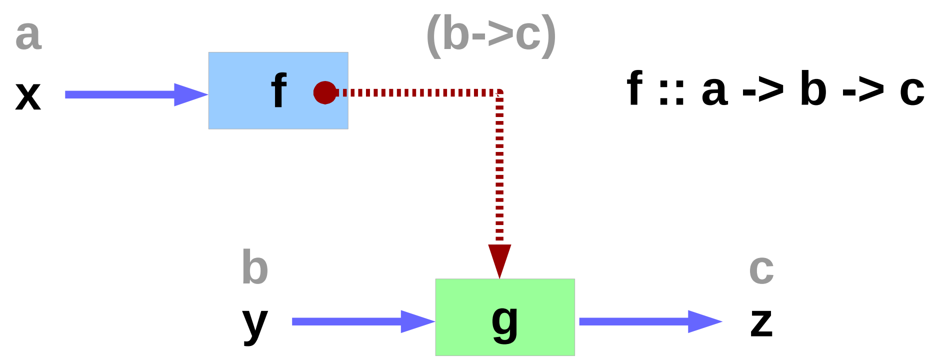
just like integers, or floating-point values, or other types.

- a function can take other functions as parameters
- a function takes a parameter and produces another function (curried function)

| | |
|---|---|
| **f x y** | **f :: a -> b -> c** |
| **(f x) y** | **f :: a -> (b -> c)** |

**f x** returns a function of type **b -> c**

| | |
|---|---|
| **g y** | **g :: b -> c** |

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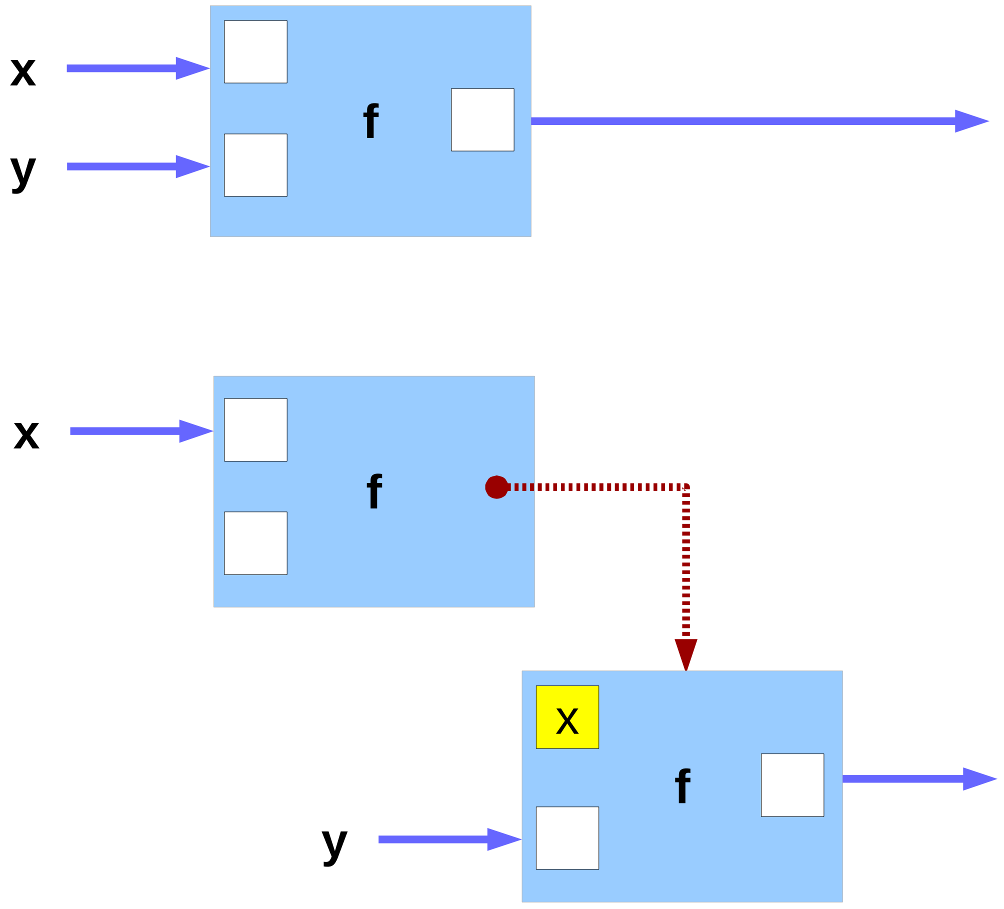
# Currying Examples




http://learnyouahaskell.com/functors-applicative-functors-and-monoids

# Uncurrying Examples

```
fn :: a -> b -> c -> d

uncurry $ fn :: (a, b) -> c -> d

uncurry . uncurry  $ fn :: (a, b, c) -> d
```

https://wiki.haskell.org/Lifting

# Polymorphic Functions

specific types vs. arbitrary types

a **polymorphic** functions – an abstract type

each type variable is generally a lower-case letter.

Example) A translate function

takes a function **f** and a distance **d**

returns a new function **g**

that is **f** "translated" **d** units to the right

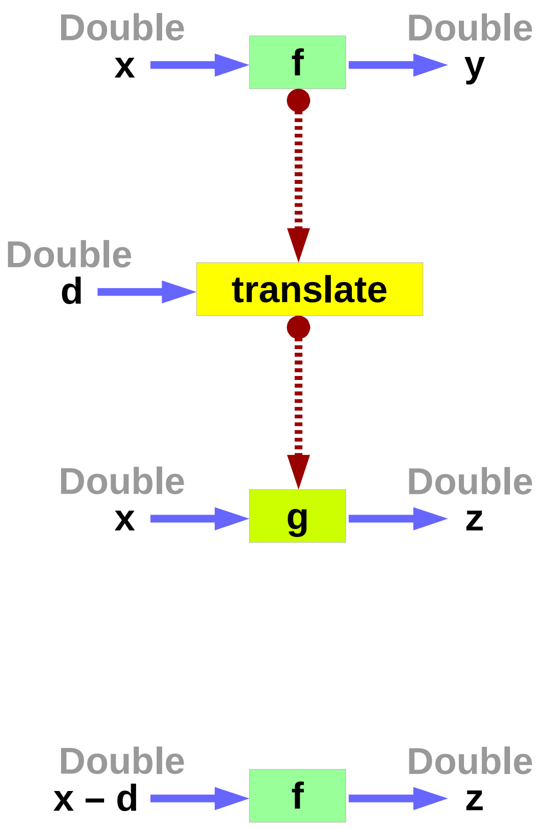
http://www.toves.org/books/hsfun/

# Polymorphic Function Examples

**translate** :: (Double -> Double) -> Double -> (Double -> Double)

**translate f** d = **g where g** x = **f** (x – d)

**translate** :: (Double -> a) -> Double -> (Double -> a)




http://www.toves.org/books/hsfun/

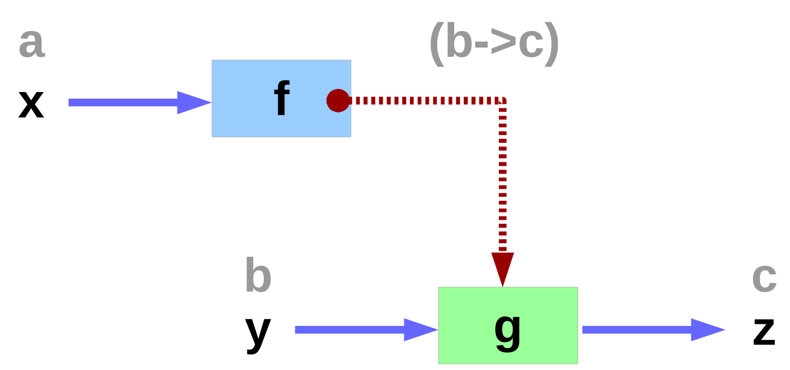
# Currying

Currying recursively transforms
a function that takes multiple arguments
into a function that takes just a single argument and
returns another function if any arguments are still needed.

**f :: a -> b -> c**

| | |
|---|---|
| **f x y** | **f :: a -> b -> c** |
| **(f x) y** | **f :: a -> (b -> c)** |
| **g y** | **g :: b -> c** |

**f :: a -> b -> c**

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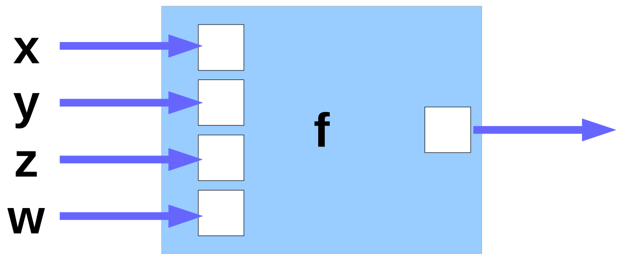
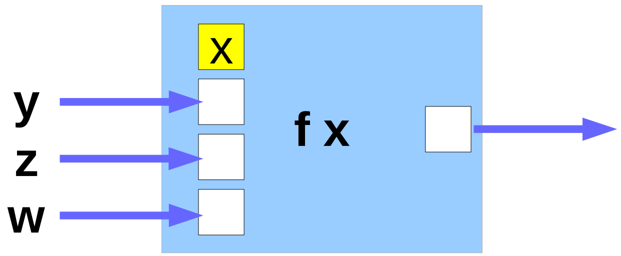
# Partially Applied Functions – f, (f x)



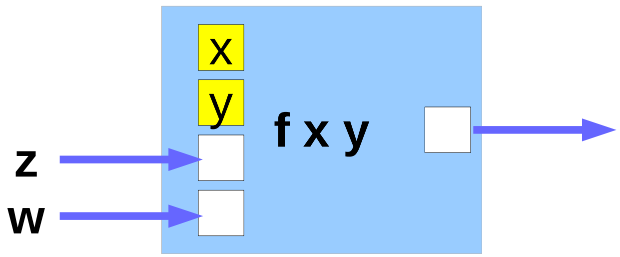

**f :: a -> b -> c -> d -> e**
**f  x  y  z  w  = …**

(f  x)  y  z  w



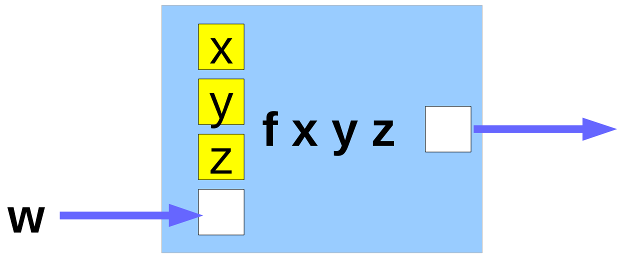

**g1 :: b -> c -> d -> e**
**g1  y  z  w  = …**

# Partially Applied Functions – (f x y), (f x y z)




(f x y) z w

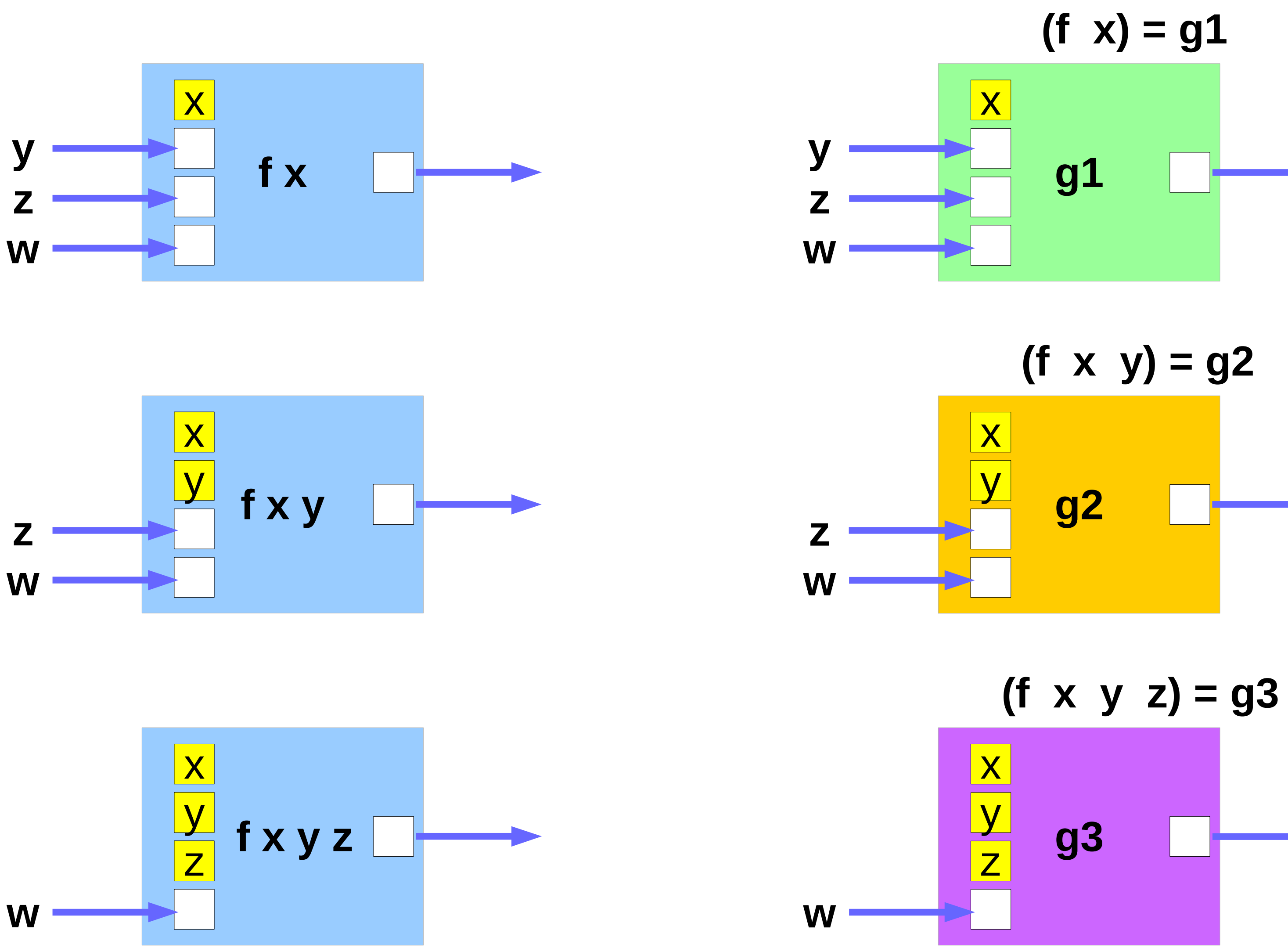
g2 :: c -> d -> e
g2 z w = …



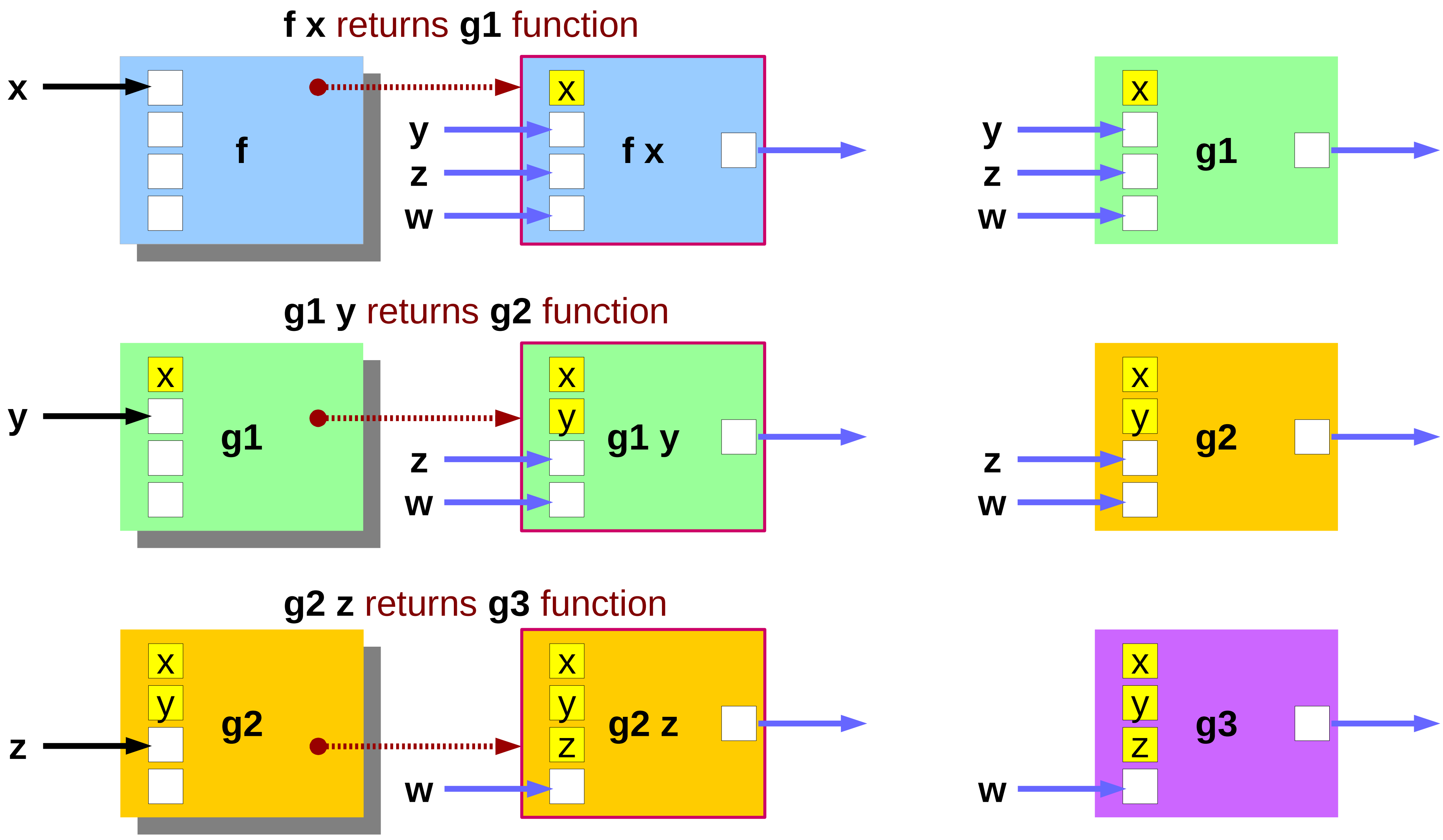

(f x y z) w

g3 :: d -> e
g3 w = …

# Partially Applied Functions – g1, g2, g3



# Returning Functions

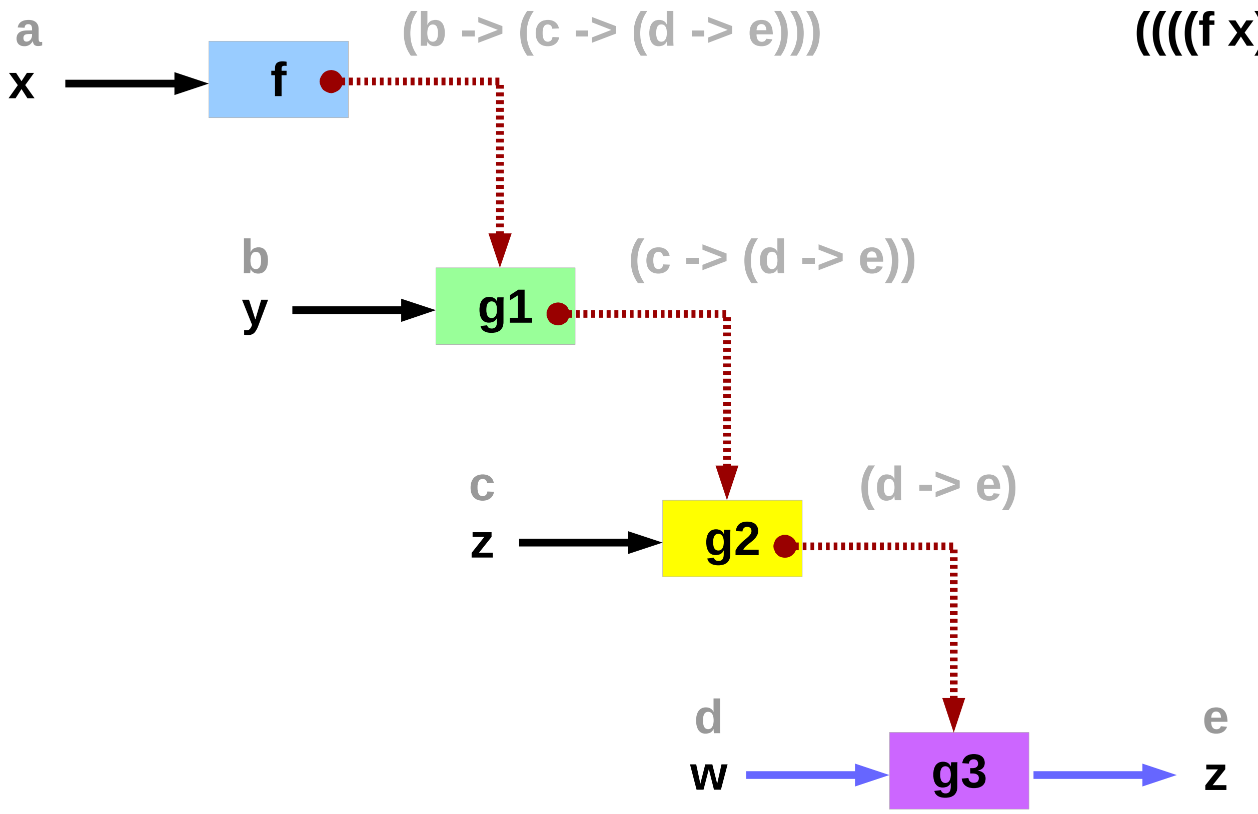


# Currying Examples

**f :: a -> b -> c -> d -> e**

**f :: a -> (b -> (c -> (d -> e)))**

**((((f x) y) z) w)**



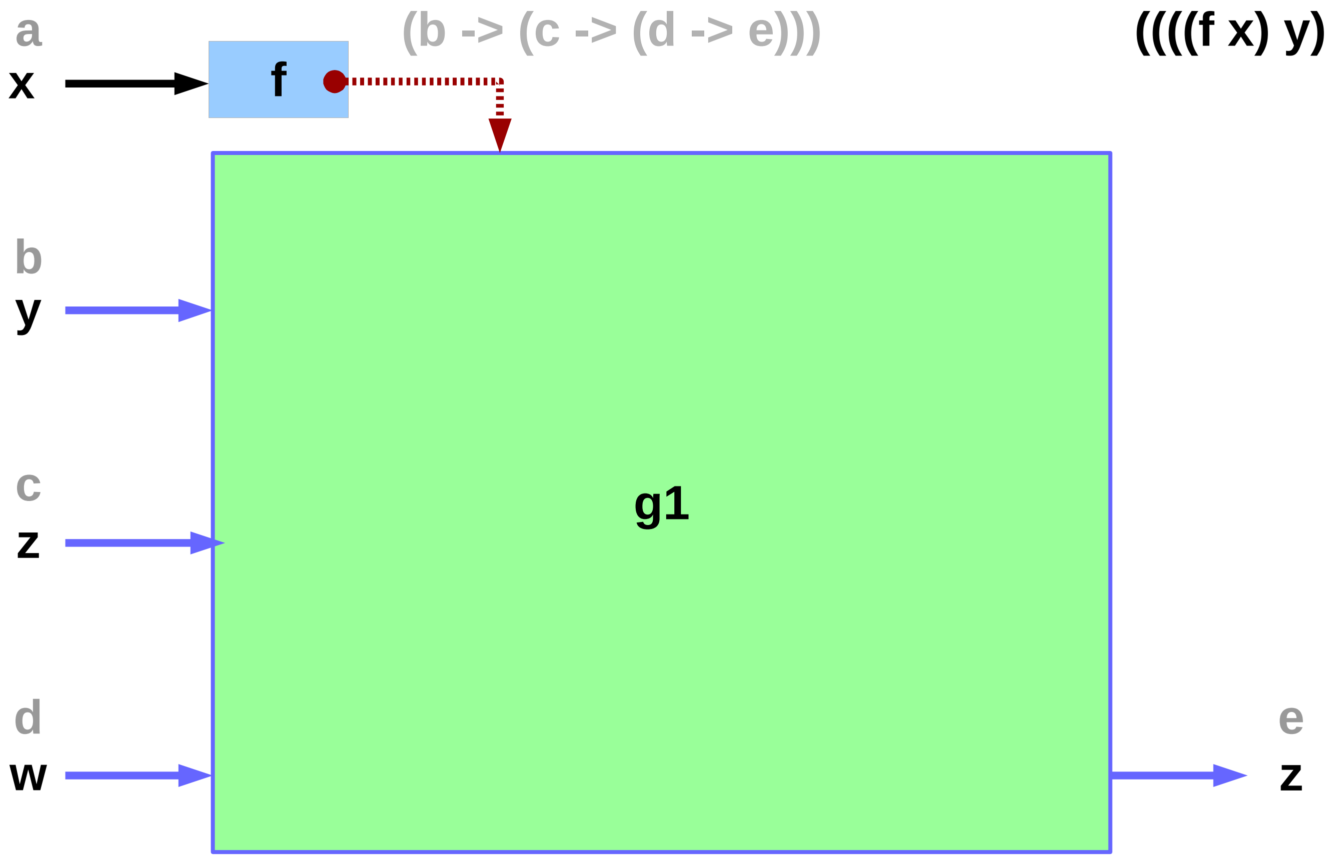

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

# Curying Examples

f :: a -> b -> c -> d -> e

f :: a -> (b -> (c -> (d -> e)))

((((f x) y) w) z)

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# Currying Examples

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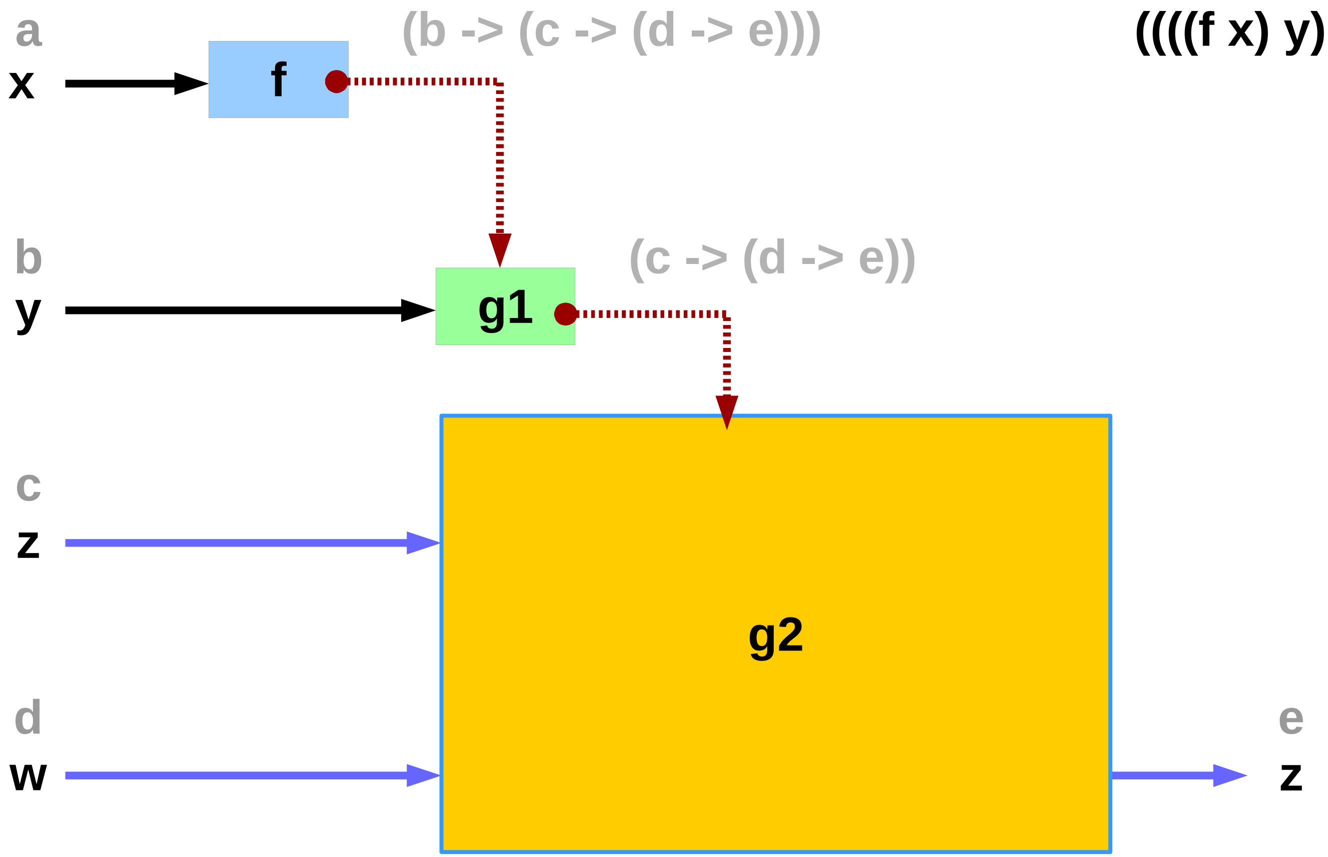


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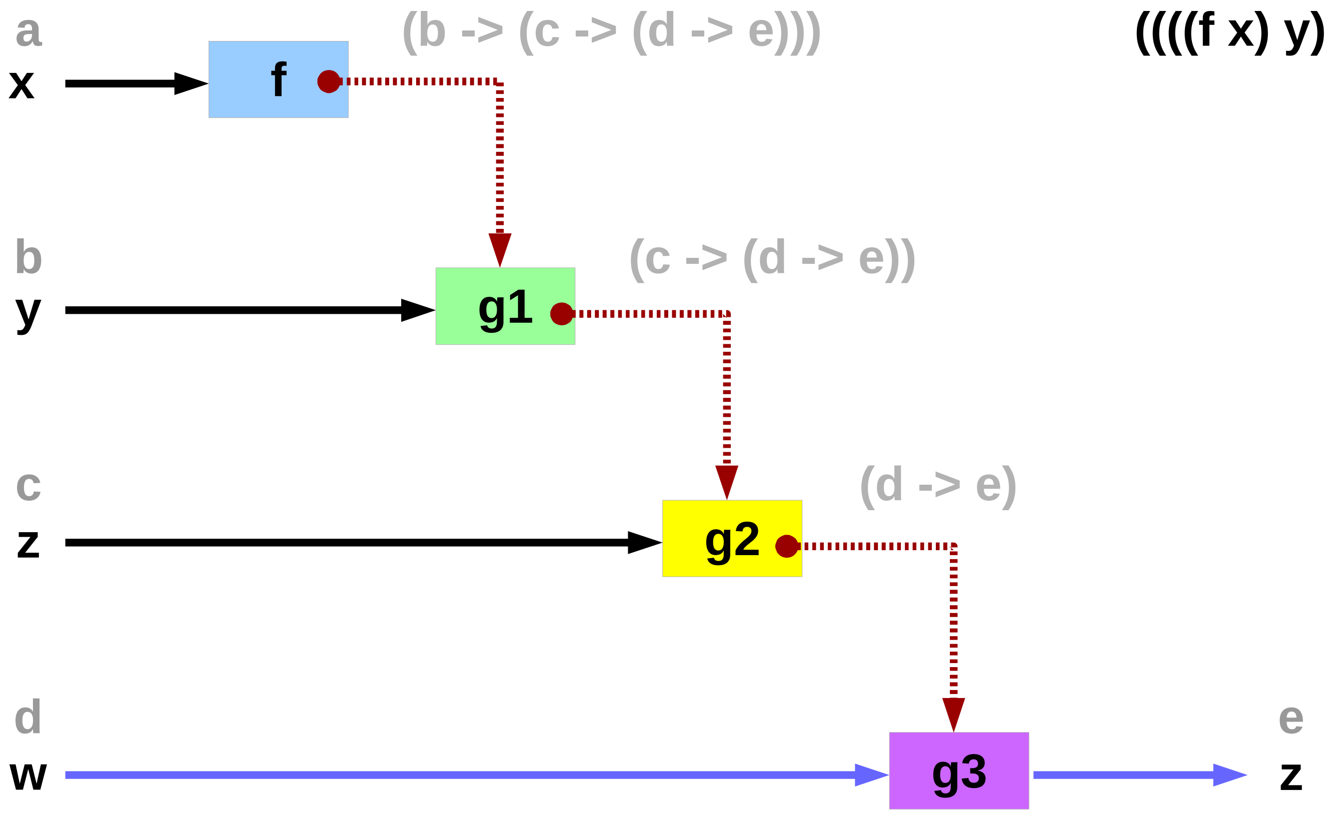


((((f x) y) z) w)

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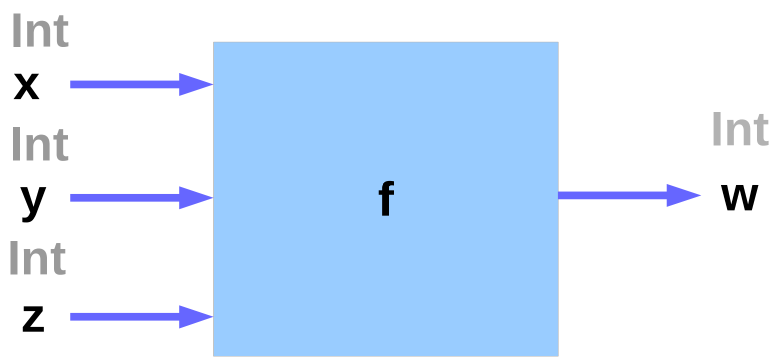
# Currying Examples

**mult :: Int -> Int -> Int -> Int**

**(((mult x) y) z)**

**f :: a -> (b -> (c -> d))**

**(((f x) y) z)**

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# Partial Applications

**mult :: Int -> Int -> Int -> Int**

**mult x y z**

**mult $a_1$ y z = g1 y z**

**mult $a_1$ $a_2$ z = g2 z**

**mult $a_1$ $a_2$ $a_3$** constants

**f :: Int -> (Int -> (Int -> Int))**

**f :: Int -> (Int -> (Int -> Int))**

**f x y z**

**f x :: Int -> (Int -> Int)**
**g1 :: Int -> (Int -> Int)**

**g1 y z**

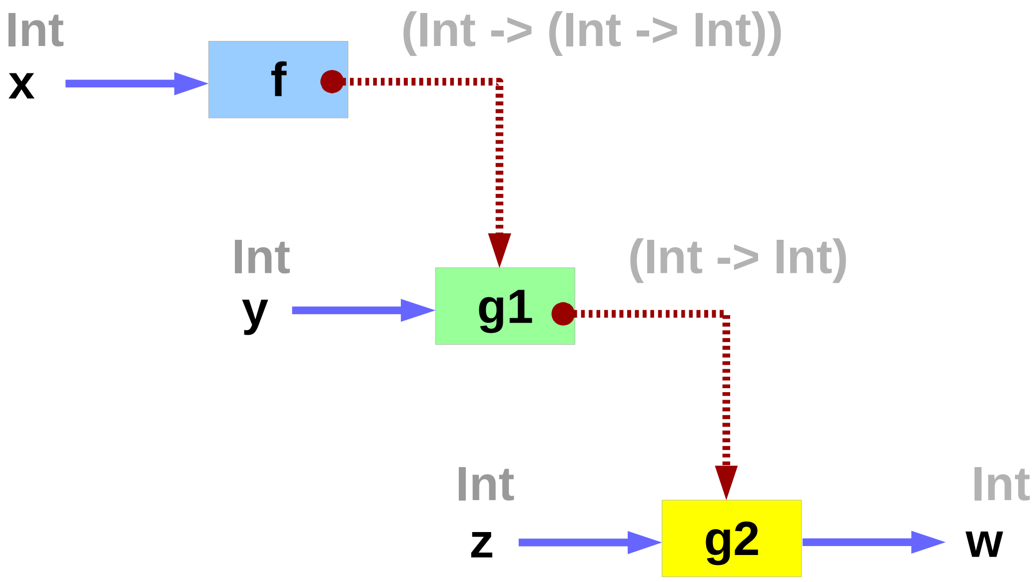
**f x y :: Int -> Int**
**g2 :: Int -> Int**

**g2 z**

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

# Returning Functions

**mult :: Int -> Int -> Int -> Int**




**mult** x y z

**mult** $a_1$ y z

**mult** $a_1$ $a_2$ z

**mult** $a_1$ $a_2$ $a_3$

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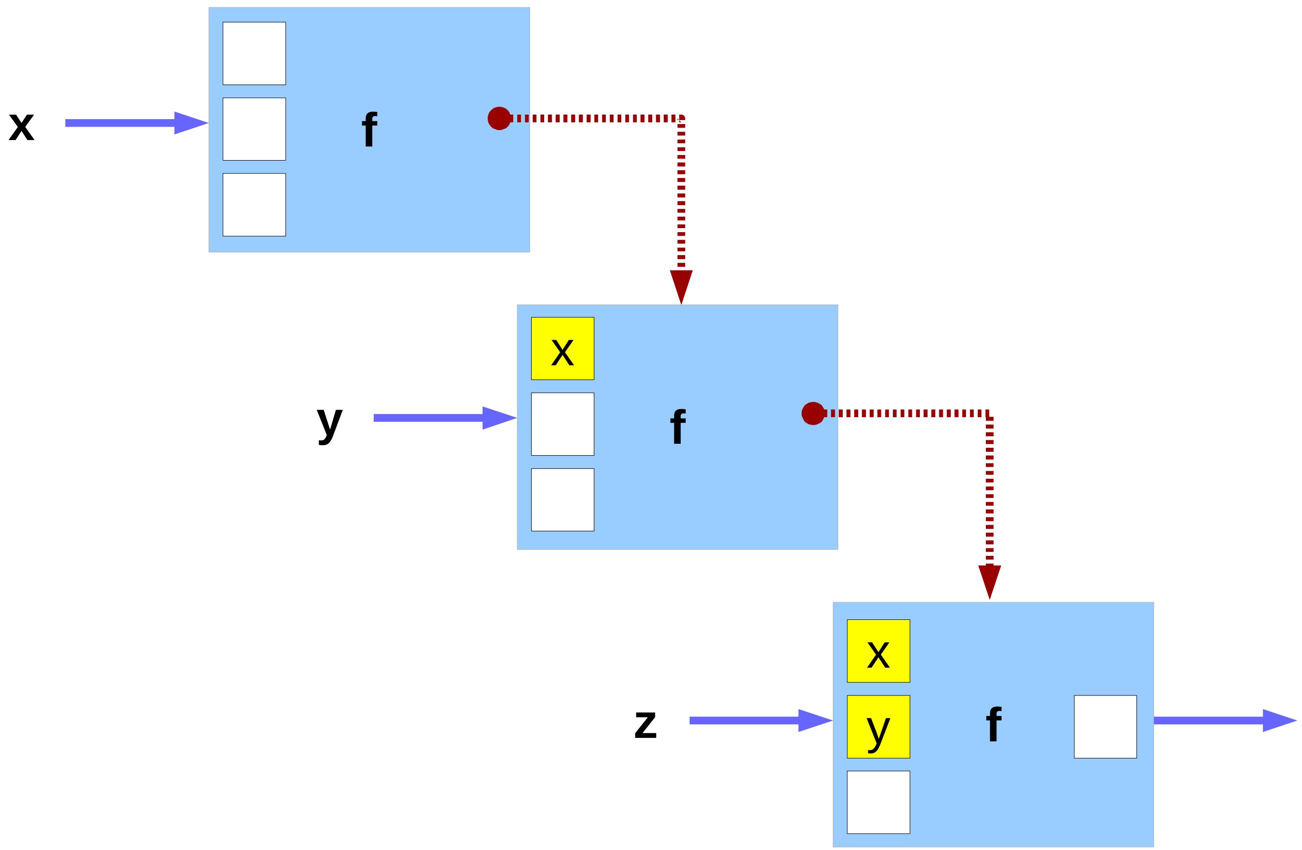
# Currying Examples

**mult :: Int -> Int -> Int -> Int**




**mult** **x** **y** **z**

**mult** $a_1$ **y** **z**

**mult** $a_1$ $a_2$ **z**

**mult** $a_1$ $a_2$ $a_3$

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

# Anonymous Function

```
\x -> x + 1
```

```
(\x -> x + 1) 4
5 :: Integer
```

```
(\x y -> x + y) 3 5
8 :: Integer
```

**Lambda Expression**

```
addOne = \x -> x + 1
```

https://wiki.haskell.org/Anonymous_function

# let … in …

```
cylinder :: (RealFloat a) => a -> a -> a
cylinder r h =
    let sideArea = 2 * pi * r * h
        topArea = pi * r ^2
    in  sideArea + 2 * topArea
```

The form is let <<u>bindings</u>> in <expression>.

The <u>names</u> that you define in the let part
are <u>**accessible**</u> to the expression after the in part.

Notice that the <u>names</u> are also aligned in a <u>single</u> <u>column</u>.

For now it just seems that let puts the <u>bindings</u> first
and the expression that uses them later
whereas where is the other way around.

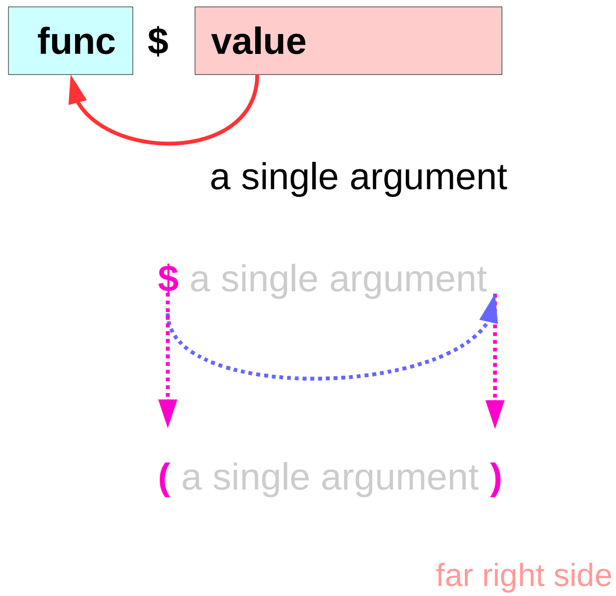
http://learnyouahaskell.com/syntax-in-functions

# $ a single argument

$ a convenience function that eliminates many parentheses.

When a **$** is encountered, the expression on its right is applied as the parameter to the function on its left.

writing an opening parentheses **(**
and then writing a closing one **)**
on the far right side of the expression.




http://learnyouahaskell.com/higher-order-functions

# $ Function Application

**($) :: (a -> b) -> a -> b**
**f $ x = f x**

f :: (a -> b) : left function
x :: a : right value
f x :: b : result

**f :: (a -> b)**
**x :: a**

**Function application** with a **space** — **f x**

- high precedence
- left-associative — **f a b c = ((f a) b) c)**

**Function application** with **$** — **f $ x**

- the lowest precedence
- right associative — **f $ a $ b $ c = f (a (b c))**

http://learnyouahaskell.com/higher-order-functions

# $ Function Application Examples

```
sum (map sqrt [1..130])

due to a low precedence
sum $ map sqrt [1..130]



sqrt 3 + 4 + 9
((sqrt 3) + (4 + 9))

sqrt (3 + 4 + 9)
sqrt $ 3 + 4 + 9
```

http://learnyouahaskell.com/higher-order-functions

# $ Right Associative Examples

because $ is right-associative

**f (g (z x))**
**f $ g $ z x**

**sum (filter (> 10) (map (*2) [2..10]))**
**sum $ filter (> 10) $ map (*2) [2..10]**

http://learnyouahaskell.com/higher-order-functions

# $ Map Function Application Examples

But apart from getting rid of parentheses,
**$** means that function application
can be treated just like another function.

map function application over a list of functions.

**map ($ 3) [(4+), (10*), (^2), sqrt]**

**[(4+ $ 3), (10* $ 3), (^2 $ 3), sqrt $ 3]**

**[7.0, 30.0, 9.0, 1.7320508075688772]**

http://learnyouahaskell.com/higher-order-functions

# **const** function

**const** x _ = x

Prelude> **const** 3 333

3

Prelude> **const** 3 99999

3

useful for passing to higher-order functions

when you don't need all their flexibility.

For example, the monadic sequence operator >>

can be defined in terms of the monadic bind operator as

x **>>** y = x **>>= const** y

(>>) = (. **const**) . (>>=)

https://stackoverflow.com/questions/7402528/whats-the-point-of-const-in-the-haskell-prelude

# **read** function

Prelude> **let x = read "True"**

Prelude> **:t x**

**x :: Read a => a**

**x** doesn't have a <u>concrete</u> type.

x is sort of an <u>expression</u>

that can provide a value of a concrete type,

when we ask for it.

ask **x** to be an **Int** or a **Bool** or anything

Prelude> **x :: Bool**

**True**

**Input: read "12"::Int**

**Output: 12**

**Input: read "12"::Double**

**Output: 12.0**

**Input: read "1.22"::Double**

**Output: 1.22**

https://stackoverflow.com/questions/7402528/whats-the-point-of-const-in-the-haskell-prelude
http://zvon.org/other/haskell/Outputprelude/read_f.html

# replicate, take, repeat, cycle, iterate

**replicate** **Int -> a -> [a]**
creates a list of **length** given by the first argument
and the items having **value** of the second argument

**take** **Int -> [a] -> [a]**
creates a list, the first argument determines,
how many **items** should be taken from the list passed
as the second argument

**repeat** **a -> [a]**
it creates an **infinite** list where all items are the first argument

**cycle** **[a] -> [a]**
it creates a **circular list** from a finite one

**Iterate** **(a -> a) -> a -> [a]**
creates an **infinite** list where the first item is calculated
by applying the function on the second argument, the second item
by applying the function on the previous result and so on.

http://zvon.org/other/haskell/Outputprelude/cycle_f.html

# replicate, take, repeat, cycle, iterate examples

**Input: replicate 3 5**
**Output: [5,5,5]**

**Input: replicate 4 "aa"**
**Output: ["aa","aa","aa","aa"]**

**Input: replicate 5 'a'**
**Output: "aaaaa"**

**Input: take 5 [1,2,3,4,5,6,7]**
**Output: [1,2,3,4,5]**

**Input: take 5 [1,2]**
**Output: [1,2]**

**Input: take 0 [1,2,3,4,5,6,7]**
**Output: []**

**Input: take 5 (repeat 3)**
**Output: [3,3,3,3,3]**

**Input: take 7 (iterate (2*) 1)**
**Output: [1,2,4,8,16,32,64]**

**Input: take 10 (cycle [1,2,3])**
**Output: [1,2,3,1,2,3,1,2,3,1]**

**Input: take 4 (repeat 3)**
**Output: [3,3,3,3]**

**Input: take 6 (repeat 'A')**
**Output: "AAAAAA"**

**Input: take 5 (repeat "A")**
**Output: ["A","A","A","A","A"]**

**Input: take 10 (cycle [1,2,3])**
**Output: [1,2,3,1,2,3,1,2,3,1]**

**Input: take 10 (cycle "ABC")**
**Output: "ABCABCABCA"**

http://zvon.org/other/haskell/Outputprelude/cycle_f.html

# flip

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y   =  f y x
```

flip f takes its (first) two arguments in the reverse order of f.

https://www.haskell.org/hoogle/?hoogle=flip

# flip

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g
  where
    g = f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g x y
  where
    g a b = f b a
```

```
flip f x y  =  g x y
flip f x  =  g x
flip f  =  g
```

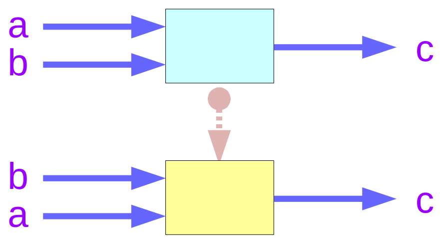
```
flip     :: (a -> b -> c) -> b -> a -> c
flip f  =  g
  where
    g a b = f b a
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f  =  g
  where
    g x y = f y x
```

https://stackoverflow.com/questions/14397128/how-does-the-flip-function-work

# flip

**flip        :: (a -> b -> c) -> b -> a -> c**
**flip f x y   =  f y x**




**flip f** takes its (first) two arguments
in the reverse order of **f**.

**f          :: (a -> b -> c)**
**flip f     :: (b -> a -> c)**

https://www.haskell.org/hoogle/?hoogle=flip

# **flip** implementation

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g
  where
    g = f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g x y
  where
    g a b = f b a
```

```
flip f x y  =  g x y
flip f x  =  g x
flip f   =  g
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g a b = f b a
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g x y = f y x
```

https://stackoverflow.com/questions/14397128/how-does-the-flip-function-work

### References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf